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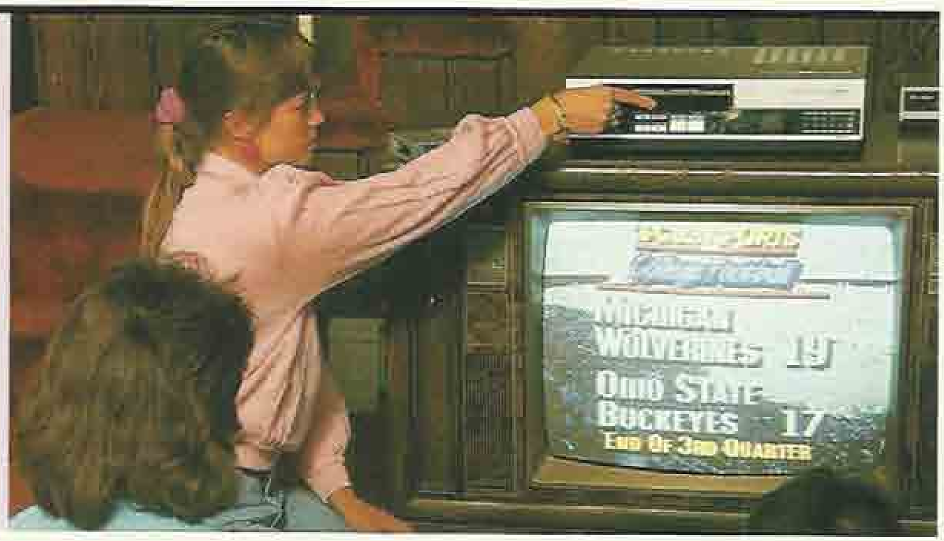
CHAPTER

10 Functions

The manufacturer of television sets has fixed costs of \$2,500,000 and an additional cost of \$350 for each set manufactured. The total cost function of manufacturing x sets is given by

$$C(x) = 2{,}500{,}000 + 350x$$

Find the cost of manufacturing 125 sets.



10–1 ■ Relations and functions

Relations

In our study of mathematics, and in the physical world, it is often useful to describe one quantity in terms of another quantity. The following examples illustrate this:

1. the demand for a product is related to the price of the product and
2. the cost to send mail is related to the weight of the piece of mail.

The relationships between these quantities are often stated as ordered pairs. Such relationships are called **relations.**

Definition of a relation

A **relation** is any set of ordered pairs.

■ **Example 10–1 A**

1. The set of ordered pairs $\{(1,2), (-3,5), (0,6), (-7,9)\}$ is a relation consisting of four ordered pairs.
2. If it costs 15¢ per mile to operate an automobile, some ordered pairs that form this relation are

   (10,1.50), (100,15), (200,30), and (500,75)

   where the first component is the miles traveled and the second component is the cost in dollars.

3. $\{(x,y)|y = 2x + 1\}$ forms a relation that contains all ordered pairs such that x is a real number and y is found by adding 1 to twice the value of x. This relation contains the following ordered pairs.

$(-4,-7), (-3,-5), (-1,-1), (0,1), (2,5)$

There are infinitely many more ordered pairs in this relation.

We see in examples 2 and 3 that the value of the second component of each ordered pair is dependent on the value given in the first component. For this reason, in example 3, for instance, we call x the **independent variable** and y the **dependent variable,** since its value is dependent on what value of x we choose. Whenever two variables are involved in the definition of a relation, one of them will be independent and the other will be dependent.

Example 10–1 B

Given that it costs 15¢ per mile to operate an automobile, a set that defines this relation is

$\{(d,C)|C = 0.15d\}$

where C is the cost in dollars to travel d miles. Then C is the dependent variable and d is the independent variable.

We give special names to the first and second components of the ordered pairs forming a relation.

Definition of domain and range

The **domain** of a relation is the set of all possible first components of the relation.
The **range** of a relation is the set of all possible second components of the relation.

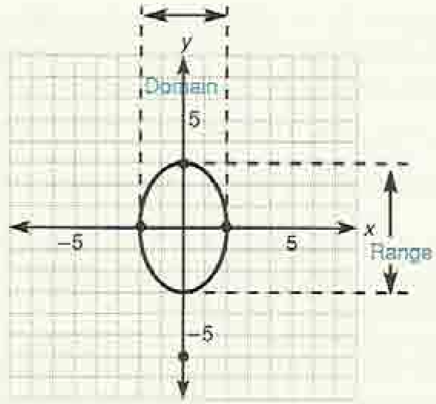
Example 10–1 C

Find the domain and range of the following relations.

1. $\{(1,2), (-3,5), (0,6), (-7,9)\}$
   The domain is $\{1,-3,0,-7\}$.
   The range is $\{2,5,6,9\}$.
2. $\{(3,-1), (5,-1), (-7,-1), (0,-1)\}$
   The domain is $\{3,5,-7,0\}$.
   The range is $\{-1\}$.

   The graph of a relation is the graph of its ordered pairs. By graphing the ordered pairs satisfying the relation, we can identify its domain and range.
3. $9x^2 + 4y^2 = 36$
   This is the equation of an ellipse whose standard form is

   $$\frac{x^2}{4} + \frac{y^2}{9} = 1$$



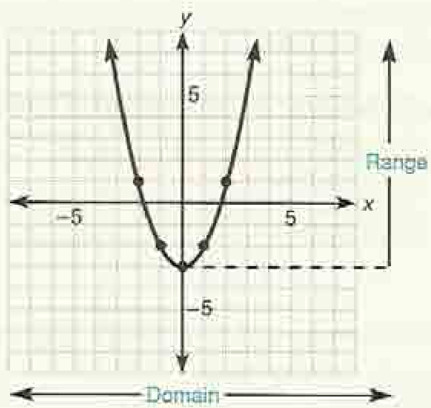
From the graph, we can see

a. the domain being the set of all possible values of x, the domain is $\{x|-2 \leq x \leq 2\}$.

b. the range being the set of all possible values of y, the range is $\{y|-3 \leq y \leq 3\}$.

4. $y = x^2 - 3$

This is the graph of a parabola with the y-axis the axis of symmetry.



From the graph, we can see that

a. x can take on any real number value, so the domain is $\{x|x \in R\}$.

b. y takes on any value that is greater than or equal to -3, so the range is $\{y|y \geq -3\}$.

▶ ***Quick check*** Find the domain and range of $\{(-6,0), (-3,-2), (0,4), (7,5)\}$. ■

Functions

In all phases of mathematics, from the most elementary to the most sophisticated, the idea of a function is a cornerstone for each mathematical development.

Consider the distance d in miles that an automobile travels in time t hours. Suppose that the automobile is traveling at an average rate of 50 miles per hour. The variables d and t are related by the equation

$$d = 50t$$

We say that "d is a function of t" since a change in the value of t will cause a change in the value of d. For example,

$$\begin{aligned}&\text{when } t = 2 \text{ hours, then } d = 50(2) = 100 \text{ miles}\\&\text{when } t = 5 \text{ hours, then } d = 50(5) = 250 \text{ miles}\\&\text{when } t = 10 \text{ hours, then } d = 50(10) = 500 \text{ miles}\end{aligned}$$

Note t is the independent variable and d is the dependent variable, since the value of d is dependent on the chosen value of t.

Notice that for any chosen value of the independent variable t, we get a *unique* (one and only one) value of the dependent variable d. For this reason, the equation (or correspondence) $d = 50t$ defines d as a function of t.

A function

Definition of a function

A **function** is a relation that associates with each first component of the ordered pairs *exactly one* value of the second component.

Concept

A function is a set of ordered pairs (a relation) in which no two distinct ordered pairs have the same first component.

The variables x and y are generally used when defining a mathematical function. By the above definition, for *y to be a function of x*, the two variables must be related so that for each value of x, there is assigned a unique (one and only one) corresponding value of y. Then **x is an element of the domain and y is an element of the range of the function.** This is a very important concept because when we use an equation to determine the outcome in a given situation, we want the equation to be of the type that produces only one answer.

Example 10–1 D

Determine if the following relations are functions.

1. $A = \{(1,2), (3,4), (-4,8), (0,-5)\}$ is a function since no two ordered pairs have the same first component.
   $B = \{(1,3), (-4,3), (9,3), (0,2)\}$ is a function since no two ordered pairs have the same first component.

   We see that three of the ordered pairs in the set B have the same second component, but this does not violate the definition of a function.

2. The relation $C = \{(3,0), (2,9), (3,-1), (-1,7)\}$ is *not* a function since the two ordered pairs $(3,0)$ and $(3,-1)$ have the same first components but different second components.

3. $\{(x,y)|y = 2x + 1\}$
This equation yields infinitely many ordered pairs. However, for each value of x we choose, we *will* get a different value of y. No two ordered pairs will have the same first component. The equation does define a function.

4. $\{(x,y)|y^2 = x + 1\}$
If we extract the roots of the equation $y^2 = x + 1$, we obtain
$$y = \pm\sqrt{x+1}$$
Let $x = 3$, then
$$y = \pm\sqrt{3+1} = \pm\sqrt{4} = \pm 2$$
We obtain two distinct ordered pairs, $(3,-2)$ and $(3,2)$, having the *same first component*. The equation *does not* define a function.

▶ ***Quick check*** Determine if the relation $\{(3,2), (3,1), (5,2)\}$ is a function. ■

Since a function defines a set of ordered pairs and is a special type of a relation, it has a *domain* and a *range*. The domain of a function must always be stated or implied by the nature of the equation defining the function. **If the domain is not stated, we will assume the domain to be the set of all real numbers for which the function is defined, or makes sense.**

■ Example 10–1 E

Determine the domain of the function defined by the following.

1. $\{(1,-5), (-2,3), (4,1), (-6,-7)\}$.
The domain (set of all first components) is $\{1,-2,4,-6\}$.

2. $\{(x,y)|3x + y = 4\}, x \in \{-3,-1,0,1,3\}$
Since replacement values of x make up the domain of a function, the domain in this case is restricted to $\{-3,-1,0,1,3\}$.

3. $\left\{(x,y)|y = \dfrac{3}{2x-1}\right\}$
We want the domain to be the set of all real numbers, unless for some reason we must restrict the values chosen for x. Thus, since division by 0 is undefined and the denominator of the equation $2x - 1 = 0$ when $x = \dfrac{1}{2}$, we determine the domain to be all real numbers except $\dfrac{1}{2}$. In set-builder notation, the
$$\text{domain} = \left\{x|x \in R, x \neq \frac{1}{2}\right\}$$

4. $\{(x,y)|y = \sqrt{x-1}\}$
Since $\sqrt{x-1}$ is a real number only when the radicand $x - 1$ is nonnegative, we must restrict the domain of the function to values of x for which $x - 1 \geq 0$; that is, $x \geq 1$. Thus the
$$\text{domain} = \{x|x \geq 1\}$$

▶ ***Quick check*** Determine the domain of the function defined by $y = \dfrac{5}{3x+2}$. ■

Finding the domain of a function

1. The domain is all real numbers except restricted numbers.
2. If the function is defined by $y = \frac{P(x)}{Q(x)}$, factor $Q(x)$, set each factor containing a variable equal to zero, and solve for the variable. This yields the values of the variable *not* in the domain of the function.
3. If the function is defined by $y = \sqrt{P(x)}$, solve the inequality $P(x) \geq 0$. This yields the values of the variable that *are* in the domain of the function.

Recall that two or more points having the same first component lie on the same vertical line. Also we have learned that a function cannot have two or more ordered pairs having the same first component (the abscissa of the point). These facts lead us to a visual test for determining if a particular graph does or does not represent a function.

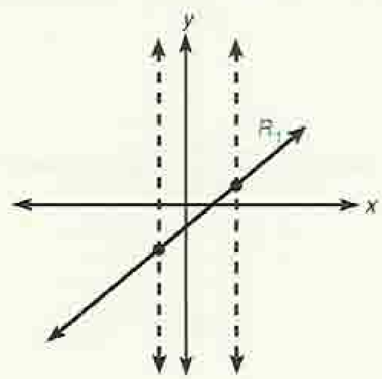
Vertical line test for a function

If every vertical line drawn in the plane intersects the graph of a relation in *at most one point,* the relation is a function.

Example 10–1 F

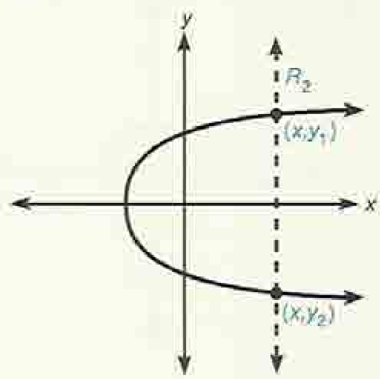
Determine by the vertical line test if the following graphs represent functions.

1.



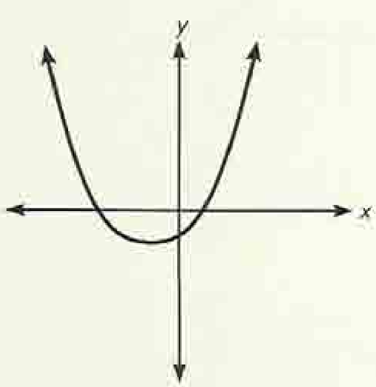
Since any vertical line drawn in the plane will intersect the graph of relation R_1 in *only one point,* R_1 is a function.

2.



Many lines can be drawn in the plane that will intersect the graph of relation R_2 *in two points,* so R_2 *is not* a function, assuming x is the independent variable.

▶ ***Quick check*** Determine by the vertical line test if the graph represents a function.



Mastery points

Can you

- Find the domain and range of a relation?
- Determine if a relation defines a function?
- Determine if a given graph of a relation represents a function by using the vertical line test?

Exercise 10–1

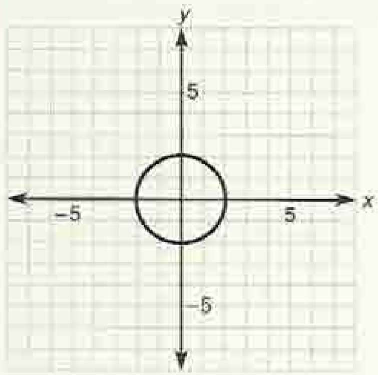
Determine the domain and range of the following relations. See example 10–1 C.

Example $\{(-6,0), (-3,-2), (0,4), (7,5)\}$

Solution Domain (set of first components) $= \{-6,-3,0,7\}$
Range (set of second components) $= \{0,-2,4,5\}$

1. $\{(8,0), (5,4), (9,3), (6,4)\}$
2. $\{(-5,4), (0,7), (-3,-2), (-2,4)\}$
3. $\{(-4,1), (1,2), (-4,3), (1,9)\}$
4. $\{(-4,3), (5,4), (-4,4), (1,1)\}$
5. $\{(6,-1), (1,1), (2,-1), (3,1)\}$
6. $\{(5,4), (7,6), (3,4), (-2,6)\}$
7. $\{(5,3), (6,3), (7,3), (6,-4)\}$
8. $\{(1,4), (2,4), (-6,4), (9,4)\}$

9.



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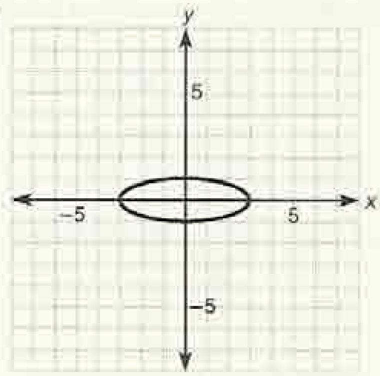

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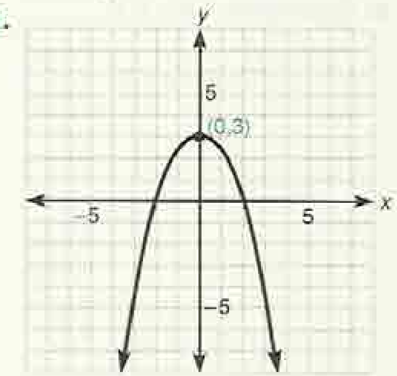

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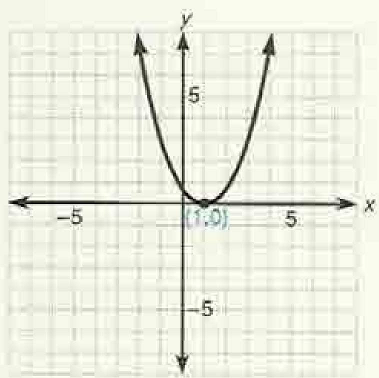

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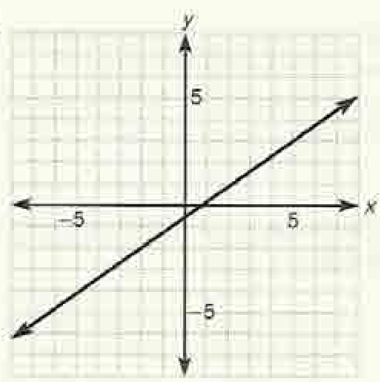

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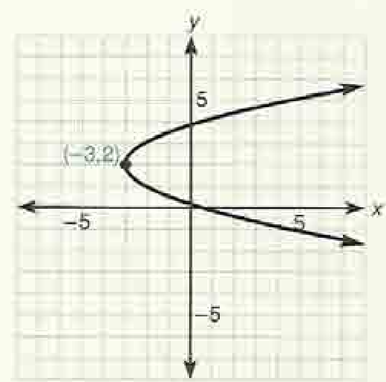

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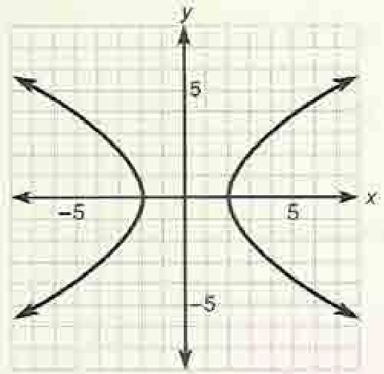

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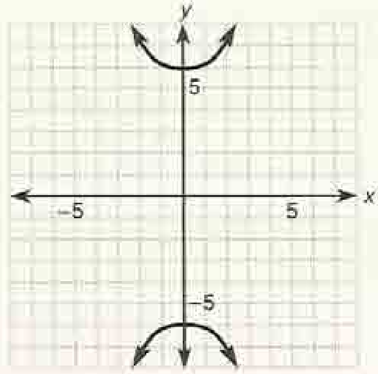


Determine whether or not the given relation defines a function. If not, show why not by an example. See example 10–1 D.

Example $\{(3,2), (3,1), (5,2)\}$

Solution Since the ordered pairs $(3,2)$ and $(3,1)$ are distinct and they both have the same first component, the relation is *not* a function.

17. $\{(1,4), (3,6), (-1,5), (8,3)\}$

18. $\{(-4,0), (0,0), (6,7), (8,-6)\}$

19. $\{(1,2), (2,3), (-1,6), (2,-7)\}$

20. $\{(-6,2), (4,7), (7,4), (-6,0)\}$

21. $\{(-1,2), (-1,6), (-1,8), (-1,0)\}$

22. $\{(1,4), (2,4), (-3,4), (-6,4)\}$

23. $\{(-3,1), (1,1), (-7,1), (9,1)\}$

24. $\{(1,1), (2,2), (3,3), (4,4)\}$

25. $\{(x,y)|y = x + 7\}$

26. $\{(x,y)|y = 3 - 4x\}$

27. $\{(x,y)|y = x^2\}$

28. $\{(x,y)|y = x^2 - x + 1\}$

29. $\{(x,y)|x = y^2\}$

30. $\{(x,y)|x = y^2 + 2\}$

31. $\{(x,y)|y = -3\}$

32. $\{(x,y)|y = 4\}$

33. $\{(x,y)|x = -10\}$

34. $\{(x,y)|x = 0\}$

35. $\{(x,y)|x - 3 = 0\}$

36. $\{(x,y)|x + 4 = 0\}$

Determine the domain of each of the given functions. Write the answers in set-builder notation where possible. See example 10–1 E.

Example $y = \dfrac{5}{3x + 2}$

Solution Since division by 0 is undefined, and since $3x + 2 = 0$ when $x = -\dfrac{2}{3}$, the domain is the set of all real numbers *except* $-\dfrac{2}{3}$. In set-builder notation,

$$\text{domain} = \left\{x|x \in R, x \neq -\frac{2}{3}\right\}$$

37. $\{(x,y)|y = 2x - 3\}; x \in \{-3,-1,0,1,3\}$

38. $\{(x,y)|y = 4 - 3x\}; x \in \{-4,-2,0,2,4\}$

39. $\{(x,y)|y = x\}; x \in \{-5,-3,0,3,5\}$

40. $\{(x,y)|y = -x\}; x \in \{-2,-1,0,7,8\}$

41. $\left\{(x,y)|y = \dfrac{1}{x}\right\}; x \in \{-5,-1,1,2,4\}$

42. $\{(x,y)|y = 4x - 3\}$

43. $\{(x,y)|x + y = 8\}$

44. $\{(x,y)|y = 3x^2\}$

45. $\{(x,y)|y = x^2 + 2x + 1\}$

46. $\{(x,y)|xy = 2\}$

47. $\left\{(x,y)|y = \frac{1}{x}\right\}$

48. $\left\{(x,y)|y = \frac{3}{x + 7}\right\}$

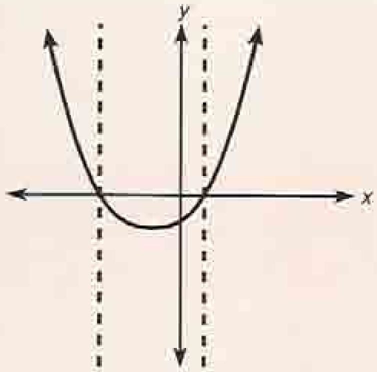
49. $\left\{(x,y)|y = \frac{5}{2x + 3}\right\}$

50. $\{(x,y)|y = \sqrt{x - 9}\}$

51. $\{(x,y)|y = \sqrt{3x + 4}\}$

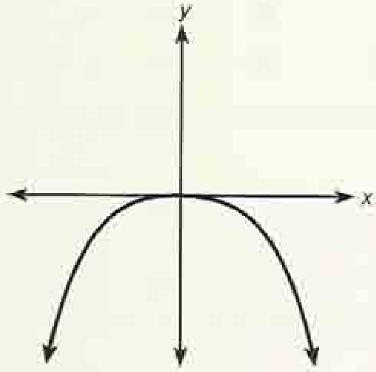
Use the vertical line test to identify which of the following graphs represent functions where y is a function of x. Explain the answers. See example 10–1 F.

Example



Solution Since any vertical line drawn in the plane will interest the graph in *only one point*, the graph is a function.

52.



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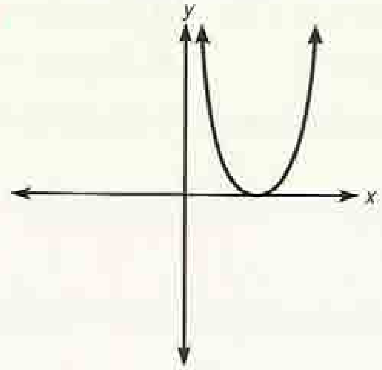

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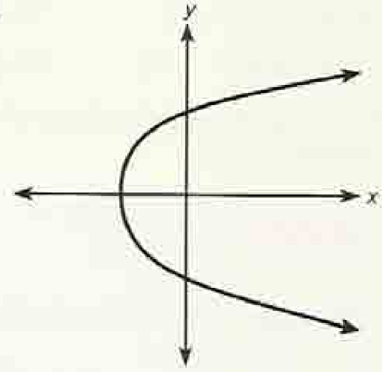

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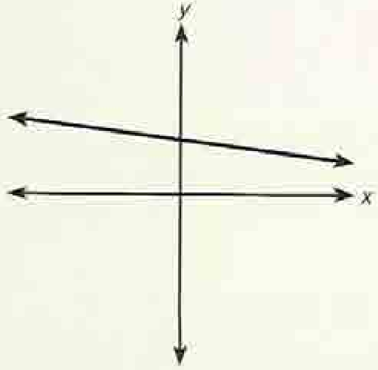

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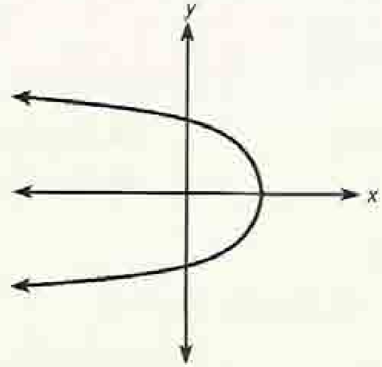

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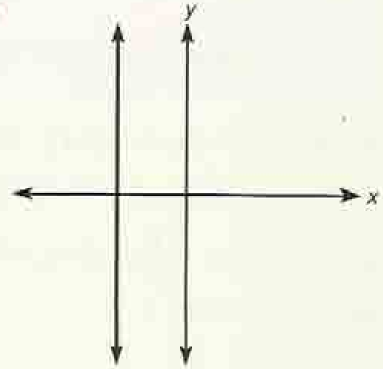

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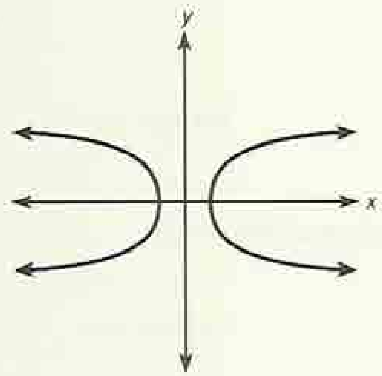

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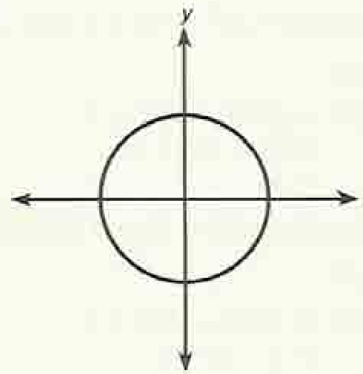

Review exercises

Simplify the following expressions. Assume all variables are nonzero. Express answers with positive exponents only. See sections 3–1 and 3–3.

1. $(2x^4y^{-3})^2$
2. $x^3 \cdot x^2 \cdot x \cdot x^0$
3. $\dfrac{x^{-2}y^3}{x^{-2}y^{-1}}$
4. Is $x^{y+1} = x^y \cdot y$? Explain. See section 3–1.
5. Factor $4x^2 - 16y^2$. See section 3–7.
6. The length of the diagonal of a square is $6\sqrt{3}$ units long. Find the length of each side. See section 6–4.
7. A rectangular field is twice as long as it is wide. Find the dimensions if the area is 7,200 square meters. See section 6–4.

10–2 ▪ Functional notation

The lowercase letters f, g, and h are commonly used to denote a function. For example, the function defined by the equation $y = 2x - 5$, where x is the independent variable, is often written $f(x) = 2x - 5$. We read the symbol "$f(x)$" as "f at x" or "f of x," which means "the value of the function at x."

Note We have replaced the dependent variable y with the symbol "$f(x)$." This new symbol represents a value of the function. Remember, $f(x)$ *does not* mean f times x.

The function has been given the name f and since $f(x)$ replaced y in the equation, $f(x)$ represents the element in the range of the function f that is associated with the chosen domain element, x.

To illustrate, given the function $y = 2x - 5$ and the domain element $x = 3$,

$$\begin{aligned} y &= 2x - 5 \\ &= 2(3) - 5 \\ &= 6 - 5 \\ &= 1 \end{aligned} \qquad \begin{aligned} f(x) &= 2x - 5 \\ f(3) &= 2(3) - 5 \\ &= 6 - 5 \\ &= 1 \end{aligned} \qquad \text{Replace } x \text{ with } 3$$

Thus, for the domain element $x = 3$, the corresponding range element is $y = f(3) = 1$ and the ordered pair $(3,1)$ is an element in the function f.

Note We read "$f(3) = 1$" as "the value of the function f when $x = 3$ is 1."

Example 10–2 A

Given $g(x) = 3x^2 - 4x + 1$, find

1. $g(3)$

$$\begin{aligned} g(3) &= 3(3)^2 - 4(3) + 1 && \text{Replace } x \text{ with } 3 \\ &= 27 - 12 + 1 \\ &= 16 \end{aligned}$$

Therefore $g(3) = 16$ and $(3,16)$ is an element of function g.

2. $g(1)$

$$\begin{aligned} g(1) &= 3(1)^2 - 4(1) + 1 && \text{Replace } x \text{ with } 1 \\ &= 3 - 4 + 1 \\ &= 0 \end{aligned}$$

$g(1) = 0$ and $(1,0)$ is an element of function g.

3. $g(a)$

$$\begin{aligned} g(a) &= 3(a)^2 - 4(a) + 1 && \text{Replace } x \text{ with } a \\ &= 3a^2 - 4a + 1 \end{aligned}$$

4. $g(x + h)$

$$\begin{aligned} g(x + h) &= 3(x + h)^2 - 4(x + h) + 1 && \text{Replace } x \text{ with } (x + h) \\ &= 3(x^2 + 2xh + h^2) - 4x - 4h + 1 \\ &= 3x^2 + 6xh + 3h^2 - 4x - 4h + 1 \end{aligned}$$

Thus, $g(x + h) = 3x^2 + 6xh + 3h^2 - 4x - 4h + 1$.

5. $g(x + h) - g(x)$

We have already determined that

$$g(x + h) = 3x^2 + 6xh + 3h^2 - 4x - 4h + 1$$
$$g(x) = 3x^2 - 4x + 1$$

Then

$$\begin{aligned} g(x + h) - g(x) \\ &= (3x^2 + 6xh + 3h^2 - 4x - 4h + 1) - (3x^2 - 4x + 1) \\ &= 3x^2 + 6xh + 3h^2 - 4x - 4h + 1 - 3x^2 + 4x - 1 && \text{Remove grouping symbols} \\ &= 6xh + 3h^2 - 4h && \text{Combine like terms} \end{aligned}$$

Therefore, $g(x + h) - g(x) = 6xh + 3h^2 - 4h$.

6. $\dfrac{g(x + h) - g(x)}{h} \quad (h \neq 0)$

We have determined that

$$g(x + h) - g(x) = 6xh + 3h^2 - 4h$$

Then

$$\begin{aligned} \frac{g(x + h) - g(x)}{h} &= \frac{6xh + 3h^2 - 4h}{h} && \text{Divide each member by } h \\ &= \frac{h(6x + 3h - 4)}{h} && \text{Factor out } h \\ &= 6x + 3h - 4 && \text{Reduce by } h \end{aligned}$$

Thus, $\dfrac{g(x + h) - g(x)}{h} = 6x + 3h - 4$.

▶ ***Quick check*** Given $f(x) = 3x^2 - 4$, find a. $f(-3)$; b. $f(x + h)$; c. $f(x + h) - f(x)$; d. $\dfrac{f(x + h) - f(x)}{h}$, $(h \neq 0)$. ■

Composite functions

When the variable in a function is replaced by another function, we form what is called a **composite function.**

■ Example 10–2 B

Given functions $f(x) = 2x + 3$ and $g(x) = 2x^2 - 1$, find the following.

1. $f[g(x)]$

Replace x with $2x^2 - 1$ in each member of the equation $f(x) = 2x + 3$.

$$\begin{aligned} f(x) &= 2x + 3 \\ f(2x^2 - 1) &= 2(2x^2 - 1) + 3 && \text{Replace } x \text{ with } 2x^2 - 1 \\ &= 4x^2 - 2 + 3 && \text{Multiply as indicated} \\ &= 4x^2 + 1 \end{aligned}$$

Thus, $f[g(x)] = 4x^2 + 1$.

2. $g[f(x)]$

Replace x with $2x + 3$ in each member of the equation $g(x) = 2x^2 - 1$.

$$\begin{aligned} g(x) &= 2x^2 - 1 \\ g(2x + 3) &= 2(2x + 3)^2 - 1 && \text{Replace } x \text{ with } 2x + 3 \\ &= 2(4x^2 + 12x + 9) - 1 && \text{Multiply as indicated} \\ &= 8x^2 + 24x + 18 - 1 \\ &= 8x^2 + 24x + 17 \end{aligned}$$

Thus, $g[f(x)] = 8x^2 + 24x + 17$.

Note Remember that a new function is formed by the composition of the two functions. The new functions (composite functions) in examples 1 and 2 are defined by $f[g(x)]$ and $g[f(x)]$. In general, $f[g(x)] \neq g[f(x)]$.

▶ ***Quick check*** Given $f(x) = x + 3$ and $g(x) = 3x^2 - 2$, find a. $f[g(x)]$; b. $g[f(x)]$. ■

When we are given a domain element, we can find the corresponding range element in the composite function as illustrated in the following examples.

■ Example 10–2 C

Given $f(x) = 3x - 1$ and $g(x) = x^2 + 5$, find the following.

1. $f[g(2)]$

$f[g(2)]$ means to evaluate $f(x)$ when $x = g(2)$. We first evaluate $g(2)$.

$$\begin{aligned} g(2) &= (2)^2 + 5 && \text{Replace } x \text{ with 2 in } g(x) = x^2 + 5 \\ &= 4 + 5 \\ &= 9 \end{aligned}$$

We now replace $g(2)$ in $f[g(2)]$ with 9.

$$f[g(2)] = f(9) = 3(9) - 1 \quad \text{Replace } x \text{ with 9 in } f(x) = 3x - 1$$
$$= 27 - 1$$
$$= 26$$

We have found that $f[g(2)] = f(9) = 26$ and the ordered pair (2,26) is in $f[g(x)]$.

2. $g[f(-3)]$

$g[f(-3)]$ means to evaluate $g(x)$ when $x = f(-3)$. We first find $f(-3)$.

$$f(-3) = 3(-3) - 1 \quad \text{Replace } x \text{ with } -3 \text{ in } f(x) = 3x - 1$$
$$= -9 - 1$$
$$= -10$$

We now evaluate $g[f(-3)]$ where $f(-3) = -10$.

$$g[f(-3)] = g(-10) \quad \text{Replace } f(-3) \text{ with } -10$$
$$= (-10)^2 + 5 \quad \text{Replace } x \text{ with } -10 \text{ in } g(x) = x^2 + 3$$
$$= 100 + 5$$
$$= 105$$

Thus, $g[f(-3)] = 105$ and the ordered pair $(-3,105)$ is in $g[f(x)]$.

▶ ***Quick check*** Given $f(x) = 2x + 3$ and $g(x) = x^2 - 4$, find $f[g(-2)]$. ■

Mastery points

Can you

- Evaluate $f(x)$ for any value of x given the function f?
- Find the composition $f[g(x)]$ and the composition $g[f(x)]$?

Exercise 10–2

Find the value of each of the following if $f(x) = 3x - 2$ and $g(x) = x^2 + 2x - 5$. In problems 1–11, state your answer as an ordered pair. See example 10–2 A.

Example Given $f(x) = 3x^2 - 4$, find $f(-3)$.

Solution

$$f(x) = 3x^2 - 4 \quad \text{Original equation}$$
$$f(-3) = 3(-3)^2 - 4 \quad \text{Replace } x \text{ with domain element } -3$$
$$= 27 - 4 \quad \text{Multiply as indicated}$$
$$= 23$$

$f(-3) = 23$ and the ordered pair $(-3,23)$ is an element of the function f.

1. $f(0)$ **2.** $f(-6)$ **3.** $f\left(\frac{2}{3}\right)$ **4.** $g(0)$ **5.** $g(7)$

6. $g(-3)$ **7.** $g\left(\frac{1}{2}\right)$ **8.** $f(a)$ **9.** $f(a + 1)$ **10.** $f\left(\frac{1}{a}\right)$

11. $f(a^2)$ **12.** $f(5) - f(2)$ **13.** $f(6) - f(-3)$ **14.** $g(3) - g(1)$ **15.** $g(4) - g(-4)$

For each of the following functions, find (a) $f(x + h)$; (b) $f(x + h) - f(x)$; (c) $\dfrac{f(x + h) - f(x)}{h}$, $h \neq 0$. See example 10–2 A–4, 5, and 6.

Example Given $f(x) = 3x^2 - 4$, find $f(x + h)$, $f(x + h) - f(x)$, and $\dfrac{f(x + h) - f(x)}{h}$, $(h \neq 0)$.

Solution

a. $f(x + h) = 3(x + h)^2 - 4$ — Replace x with $x + h$

$= 3(x^2 + 2hx + h^2) - 4$ — Square the binomial

$= 3x^2 + 6hx + 3h^2 - 4$ — Distributive property

b. $f(x + h) - f(x) = (3x^2 + 6hx + 3h^2 - 4) - (3x^2 - 4)$ — Substitute

$= 3x^2 + 6hx + 3h^2 - 4 - 3x^2 + 4$ — Remove parentheses

$= 6hx + 3h^2$ — Combine like terms

c. $\dfrac{f(x + h) - f(x)}{h} = \dfrac{6hx + 3h^2}{h}$ — Substitute

$= \dfrac{h(6x + 3h)}{h}$ — Factor h from each term

$= 6x + 3h$ — Reduce to lowest terms

16. $f(x) = 4x - 1$
17. $f(x) = 4x^2$
18. $f(x) = 3x^2 - 2x$
19. $f(x) = 2x^2 + 3x + 2$
20. $f(x) = 5$
21. $f(x) = 5 - 2x$

Find the indicated values of each given function. Write the answers as ordered pairs. See example 10–2 A.

22. $f(x) = 5x - 6$; find $f(-2)$, $f(0)$, $f(2)$
23. $f(x) = 3x - 2$; find $f(-5)$, $f(0)$, $f\left(\frac{2}{3}\right)$
24. $h(x) = x^2 - 5$; find $h(-5)$, $h(0)$, $h(\sqrt{5})$
25. $h(x) = 3x^2 - 2x + 1$; find $h\left(-\frac{1}{2}\right)$, $h(0)$, $h(3)$
26. $g(x) = -7$; find $g\left(-\frac{7}{8}\right)$, $g(0)$, $g(25)$
27. $g(x) = 10$; find $g(-15)$, $g(0)$, $g\left(\frac{6}{5}\right)$
28. $h(x) = x^3 + 4$; find $h(-5)$, $h(0)$, $h\left(\frac{1}{2}\right)$

Given $f(x) = 3x^2 - 5$ and $g(x) = 4x + 1$, find the following compositions. See example 10–2 B.

Example Given $f(x) = x + 3$ and $g(x) = 3x^2 - 2$, find (a) $f[g(x)]$, (b) $g[f(x)]$.

Solution

a. $f[g(x)] = f(3x^2 - 2)$ — Replace $g(x)$ with $3x^2 - 2$

$= (3x^2 - 2) + 3$ — Replace x with $3x^2 - 2$ in $x + 3$

$= 3x^2 + 1$ — Combine

b. $g[f(x)] = g(x + 3)$ — Replace $f(x)$ with $x + 3$

$= 3(x + 3)^2 - 2$ — Replace x with $x + 3$ in $3x^2 - 2$

$= 3(x^2 + 6x + 9) - 2$ — Multiply as indicated

$= 3x^2 + 18x + 27 - 2$

$= 3x^2 + 18x + 25$

29. $f[g(x)]$
30. $g[f(x)]$
31. $f[f(x)]$
32. $g[g(x)]$

Given $f(x) = 3x^2 - 5$ and $g(x) = 4x + 2$, find the following compositions. See example 10–2 C.

Example Given $f(x) = 2x + 3$ and $g(x) = x^2 - 4$, find $f[g(-2)]$.

Solution We first evaluate $g(-2)$.

$$\begin{aligned} g(x) &= x^2 - 4 \\ g(-2) &= (-2)^2 - 4 && \text{Replace } x \text{ with } -2 \\ &= 4 - 4 = 0 \end{aligned}$$

$$\begin{aligned} f[g(-2)] &= f(0) && \text{Replace } g(-2) \text{ with 0 in } f[g(-2)] \\ &= 2(0) + 3 && \text{Replace } x \text{ with 0 in } f(x) = 2x + 3 \\ &= 3 \end{aligned}$$

The composition $f[g(-2)] = 3$.

33. $f[g(-1)]$ $\boxed{34.}$ $f[g(4)]$ 35. $g[f(1)]$ 36. $g[f(-2)]$

Solve the following word problems.

$\boxed{37.}$ The temperature in degrees Celsius C can be expressed as a function of degrees Fahrenheit F by $C = g(F)$. If $g(F) = \dfrac{5}{9}(F - 32)$, find $g(14)$, $g(32)$, and $g(212)$. Express the answers as ordered pairs $(F,g(F))$.

38. The temperature in degrees Fahrenheit F can be expressed as a function of degrees Celsius C by $F = f(C)$. If $f(C) = \dfrac{9}{5}C + 32$, find $f(0)$, $f(100)$, and $f(-10)$. Express the answers as ordered pairs $(C,f(C))$.

39. The volume V of a cube can be expressed as a function of its side s by $V = f(s) = s^3$. Find $f(1)$, $f(3)$, and $f(5)$. Write the answers as ordered pairs $(s,f(s))$. What is the domain of f?

40. The area A of a circle can be expressed as a function of its radius r by $A = g(r) = \pi r^2$, where π is a constant. Find $g\left(\dfrac{1}{2}\right)$, $g(2.1)$, and $g(8)$. (Leave the answers in terms of π.) Write the answers as ordered pairs $(r,g(r))$. What is the domain of g?

41. The cost C of sending a first-class letter can be expressed as a function of its weight w in ounces by $C = h(w) = 29 + 20(w - 1)$. Find $h(2)$, $h(3)$, and $h(5)$. Express the answers as ordered pairs $(w,h(w))$.

42. The cost C in dollars of gasoline can be expressed as a function of the number of gallons n by $C = f(n) = 0.96n$. Find $f(10)$, $f(12)$, and $f(25)$. Express the answers as ordered pairs $(n,f(n))$.

Review exercises

1. Subtract $(4x^2 + 6x - 9) - (-2x^2 - x + 5)$. See section 1–6.

2. Given $P(x) = 2x + 1$, $Q(x) = x^2 - 1$, and $R(x) = 5x - 4$, find $P(x) - Q(x) - R(x)$. See section 1–5.

3. Find the coordinates of the vertex and the x- and y-intercepts of the parabola $y = x^2 - 4x - 4$. See section 9–1.

4. Find the slope and y-intercept of the linear equation $4x - 5y = -20$. See section 7–3.

5. What is the degree of the polynomial $5x^4 - 3x^3 + 1$? See section 1–5.

6. Divide $(3x^3 - 4x^2 + 2x + 3) \div (x - 1)$. See section 4–5.


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10–3 ■ Special functions and their graphs

In previous chapters, we discussed and graphed such equations as

$$(1)\ y = 3x + 2 \quad \text{and} \quad (2)\ y = x^2 + 4x - 3$$

We found that the graph of (1) was a straight line while the graph of (2) was a parabola. These were called linear equations and quadratic equations, respectively. In this section, we will classify these as special functions and also study two other types of functions.

The linear function

The function that yields a straight line graph (except for vertical lines) will define **linear functions.**

Linear function

A function that can be written in the form

$$f(x) = mx + b$$

where m and b are real numbers, is called a **linear function.**

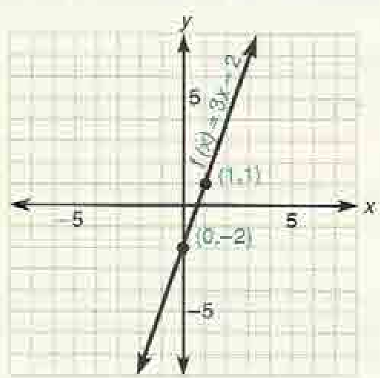
Recall in section 7–3 we found that in the linear equation $y = mx + b$, m was the slope of the line and b was the y-intercept. In like fashion, in the linear function $f(x) = mx + b$, m and b define the slope and y-intercept, respectively, of the graph of function f and they can be used to graph a linear function.

■ Example 10–3 A

Write the equation $y = 3x - 2$ as a function and graph.
Replacing y with $f(x)$, we have

$f(x) = 3x - 2$

Then $m = 3 = \frac{3}{1}$ and b (y-intercept) $= -2$. From the y-intercept, the point $(0,-2)$, move 1 unit to the right and 3 units up to obtain a second point. Draw the line through these two points.



▶ ***Quick check*** Write $y = -4x + 1$ as a function and graph the function. ■

The constant function

Recall that the graph of any equation of the form $y = k$, where k is a real number constant, is a horizontal straight line. Because the value of y is constant (always the same) for any value of x that is chosen, this equation defines a special linear function called a **constant function.**

Constant function

Any function f that can be written in the form

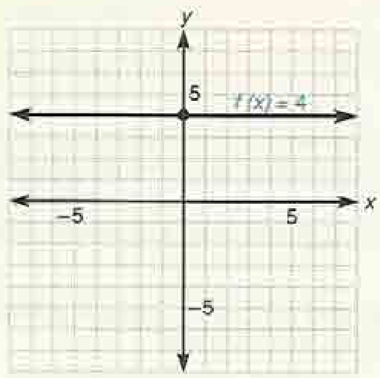
$$f(x) = k$$

where k is a real number, is called a **constant function.**

■ Example 10–3 B

Write the equation $y - 4 = 0$ as a function and graph the function.

If we solve this equation for y, we obtain $y = 4$. Then $f(x) = 4$. The graph is a horizontal straight line through the point (0,4).



■

The quadratic function

We found in section 9–1 that the graph of the quadratic equation $y = ax^2 + bx + c$ is a parabola that opens up when $a > 0$ and opens down when $a < 0$. We now write the equation using functional notation.

Quadratic function

Any function f that can be written in the form

$$f(x) = ax^2 + bx + c$$

where a, b, and c are real numbers, $a \neq 0$, is called a **quadratic function.**

Note The quadratic equation $x = ay^2 + by + c$ *does not* define a function since many vertical lines will intersect the graph in *two* points, thus failing the vertical line test. (See example 10–1 F–2.)

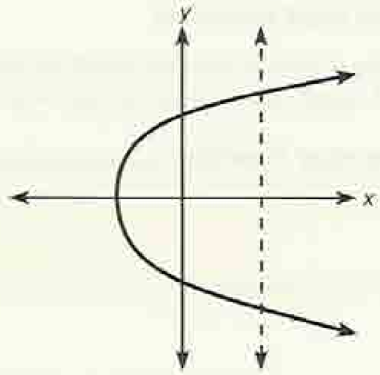


Figure 10–1

Example 10–3 C

Write the equation $y = x^2 + 3x$ as a function and graph.

$f(x) = x^2 + 3x$ — Replace y with $f(x)$

1. Let $f(x) = 0$, then

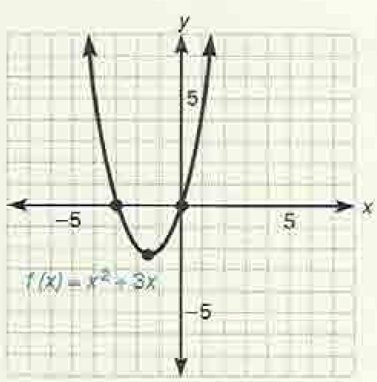
$(0) = x^2 + 3x$ — Replace $f(x)$ with 0
$\quad = x(x + 3)$ — Factor the right member
$x = 0 \quad \text{or} \quad x + 3 = 0$ — Set each factor equal to 0
$\qquad\qquad x = -3$

The x-intercepts are the points (0,0) and $(-3,0)$.

2. Let $x = 0$, then

$f(0) = (0)^2 + 3(0)$ — Replace x with 0
$\quad = 0$

The y-intercept is the point (0,0).



3. We now find the vertex.

$y = (x^2 + 3x + \quad) - (\quad)$

$y = \left(x^2 + 3x + \frac{9}{4}\right) - \left(\frac{9}{4}\right)$ — Complete the square

$y = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4}$ — Write as the square of a binomial

$y = \left[x - \left(-\frac{3}{2}\right)\right]^2 + \left(-\frac{9}{4}\right)$ — Write it in the form $y = a(x - h)^2 + k$

The vertex is the point $\left(-\frac{3}{2}, -\frac{9}{4}\right)$. ■

The linear and quadratic functions are special cases of a more general class of functions called the **polynomial function.** A polynomial function is any function defined by a polynomial.

1. $f(x) = mx + b$ (linear function) is a polynomial function of degree 1. $(m \neq 0)$
2. $f(x) = ax^2 + bx + c$ (quadratic function) is a polynomial function of degree 2. $(a \neq 0)$
3. $f(x) = ax^3 + bx^2 + cx + d$ is a polynomial function of degree 3. $(a \neq 0)$

The square root function

We now define a special function called the **square root function.** It is defined by the principal square root of a polynomial expression.

Square root function

Any function that can be stated in the form

$$f(x) = \sqrt{P}$$

where P is a polynomial in x, $P \geq 0$, is called a **square root function.**

■ Example 10–3 D

Graph the square root function $f(x) = \sqrt{x - 3}$.

Since the radicand $x - 3$ must be greater than or equal to zero, we solve the inequality $x - 3 \geq 0$ to determine the domain of f.

$x - 3 \geq 0$ Set the radicand ≥ 0

$x \geq 3$ Solve the inequality

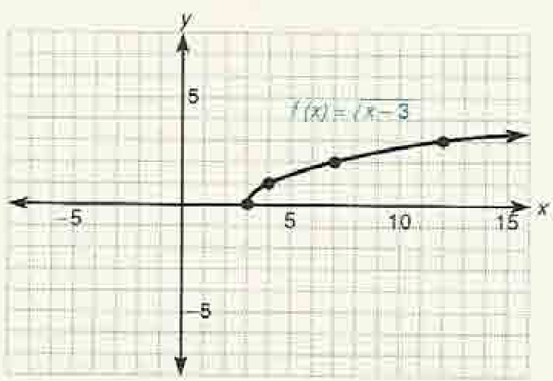
The domain is all $x \geq 3$ so we choose values of x accordingly. See the following table.

x	3	4	7	12
$f(x) = \sqrt{x - 3}$	0	1	2	3

Thus, the ordered pairs (3,0), (4,1), (7,2), and (12,3) are in f.

Note Since we can choose any value for $x \geq 3$, it is convenient to choose values of x such that $x - 3$ is a perfect square. We have done this in the table above.

If we plot a number of values of x and their corresponding values of $f(x)$, we obtain the following graph.



■

Mastery points

Can you

- Identify a linear function, a constant function, a quadratic function, and a square root function?
- Graph each of the special functions stated above?

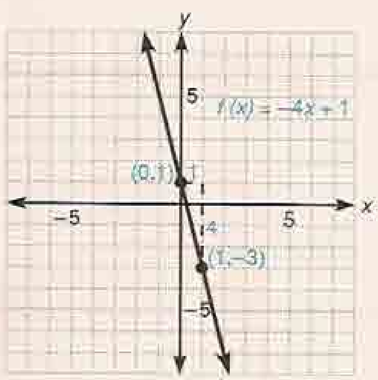
Exercise 10–3

Identify each function by name and sketch its graph. See example 10–3 A, B, C, and D.

Example $y = -4x + 1$

Solution Replacing y with $f(x)$, $f(x) = -4x + 1$, which is of the form $f(x) = mx + b$, so the function is linear.

1. $b = 1$, so the y-intercept is the point (0,1).
2. From (0,1), since $m = -4 = \dfrac{-4}{1}$, we move 1 unit to the *right* (horizontal distance) and then 4 units *down* (vertical distance) to obtain a second point, $(1,-3)$.



1. $f(x) = x + 5$
2. $f(x) = x - 2$
3. $f(x) = 4 - 3x$
4. $f(x) = 5 - x$
5. $f(x) = 6x - 1$
6. $f(x) = 2x + 3$
7. $f(x) = \frac{1}{2}x + 2$
8. $f(x) = \frac{3}{4}x - 1$
9. $f(x) = -\frac{2}{3}x - 3$
10. $f(x) = -\frac{5}{2}x + 4$
11. $f(x) = 2$
12. $f(x) = -3$
13. $f(x) = 0$
14. $f(x) = (x - 1)^2$
15. $f(x) = (x + 3)^2$
16. $f(x) = x^2 - 4x$
17. $f(x) = 5x^2 + 2x$
18. $f(x) = x^2 - x - 6$
19. $f(x) = x^2 + 8x + 12$
20. $f(x) = -x^2 - 6x - 7$
21. $f(x) = -x^2 + 2x + 15$
22. $f(x) = 2x^2 - x - 3$
23. $f(x) = -3x^2 + 2x + 1$
24. $f(x) = \sqrt{x - 5}$
25. $f(x) = \sqrt{4 - x}$
26. $f(x) = \sqrt{2 - x}$
27. $f(x) = \sqrt{4x - 1}$
28. $f(x) = \sqrt{2x + 3}$
29. $f(x) = \sqrt{x} + 5$
30. $f(x) = \sqrt{x} - 1$
31. $f(x) = -\sqrt{x + 7}$
32. $f(x) = -\sqrt{x - 6}$

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For the following functions, find $f(-2)$, $f(-1)$, $f(0)$, $f(1)$, and $f(2)$. Plot the points and connect them to graph each function. These are examples of the *absolute value function.*

33. $f(x) = |x|$
34. $f(x) = -|x|$
$\boxed{35.}$ $f(x) = |x| - 1$
36. $f(x) = |x| + 3$
37. $f(x) = 5 - |x|$
38. $f(x) = 3 - |x|$
39. Given $f(x) = |x - 2|$, find values of $f(x)$ when x takes on values -1, 0, 1, 2, 3, and 4. Plot the points and connect them to get the graph of $f(x) = |x - 2|$.
40. Graph $f(x) = -|x - 2|$ using the same values of x as in exercise 39.

Review exercises

1. Simplify the complex rational expression. See section 4–4.
$$\frac{\dfrac{25a^2 - b^2}{4a}}{\dfrac{5a + b}{7a}}$$
2. Find the solution set of the system of linear equations.
$$\begin{aligned} 3x - 4y &= 2 \\ 2x + y &= -1 \end{aligned}$$
See section 8–1.
3. Solve the inequality $z^2 - 2z \geq 3$. See section 6–7.
4. Reduce $\dfrac{3x^2 - 12}{4x^2 - 16}$ to lowest terms. See section 4–1.
5. Simplify the expression $\sqrt{\dfrac{16}{5}}$. See section 5–3.

Perform the indicated operations. See sections 5–6 and 5–7.

6. $(3\sqrt{2} - 2\sqrt{3})^2$
7. $(4 - 3i)(4 + 3i)$

10–4 ■ Inverse functions

Consider the function f defined by $f(x) = 3x - 5$. If we choose to let x take on the values -4, -2, 0, 2, and 4, we can determine that the ordered pairs

$$(-4,-17),\ (-2,-11),\ (0,-5),\ (2,1),\ \text{and } (4,7)$$

belong to the function f. Now consider the function g defined by

$$g(x) = \frac{x + 5}{3}$$

If we let x take on the values -17, -11, -5, 1, and 7 (range elements of f), we substitute to find that

1. $g(-17) = \dfrac{(-17) + 5}{3} = \dfrac{-12}{3} = -4$
2. $g(-11) = \dfrac{(-11) + 5}{3} = \dfrac{-6}{3} = -2$
3. $g(-5) = \dfrac{(-5) + 5}{3} = \dfrac{0}{3} = 0$

$$4.\ g(1) = \frac{(1) + 5}{3} = \frac{6}{3} = 2$$

$$5.\ g(7) = \frac{(7) + 5}{3} = \frac{12}{3} = 4.$$

Thus the ordered pairs

$$(-17,-4),\ (-11,-2),\ (-5,0),\ (1,2),\ \text{and}\ (7,4)$$

are elements of the function g.

Observing the functions f and g, we see that when the components of the ordered pairs in f are interchanged, we have the ordered pairs in function g. If we continued to find other ordered pairs in the two functions, we would find that the same relationship would hold. When this relationship exists between two sets of ordered pairs defining functions, we say the two functions are **inverses** of one another. Thus function g is the inverse of function f, and function f is the inverse of function g.

Given a function f, we denote the inverse of f by the symbol "f^{-1}" (read "the inverse of f" or "f inverse"). Thus $g = f^{-1}$ and $f = g^{-1}$ in the previous examples.

Note The -1 in the symbol "f^{-1}" *should not* be interpreted as an exponent. Rather, it is a necessary part of the symbol as a whole and denotes the inverse of the function.

It is important to note that the inverse of a function is not necessarily a function. Given a function defined by a set of ordered pairs, we can determine if the function has an inverse function by interchanging components.

Example 10–4 A

Determine if each function has an inverse function.

1. Given function f defined by

$f = \{(1,2), (4,5), (6,7), (-1,4)\}$

For every second component, there corresponds only one first component. Thus the inverse of f is a function and f^{-1} is defined by

$f^{-1} = \{(2,1), (5,4), (7,6), (4,-1)\}$

2. Given function g defined by

$g = \{(1,2), (2,2), (3,-6), (4,-6)\}$

If we interchange range and domain, we obtain the set of ordered pairs

$\{(2,1), (2,2), (-6,3), (-6,4)\}$

which *does not* define a function since there are two distinct pairs of ordered pairs that have the same first component, 2 and -6.

We then conclude that the inverse of a function is also a function if for each value of y there is *exactly one* value of x. We call this a **one-to-one function.**

Definition of a one-to-one function

A **one-to-one function** is any function that associates a unique value of x with each value of y.

Concept

A function is a one-to-one function if no two ordered pairs in the function have the *same second component*.

Thus the inverse of a function will be a function provided the function itself is one-to-one.

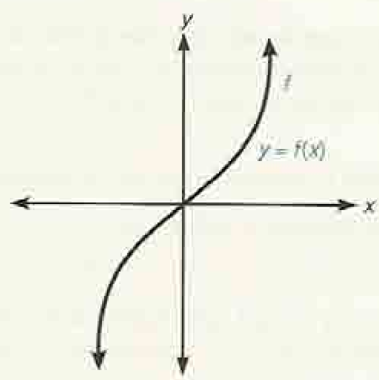
Note We have y as a function of x if for each x there is a unique y, and the function is one-to-one if for each y there is a unique x.

Recall that we have the vertical line test to determine if a graph represents a function. Since any two ordered pairs having the same second component will lie on a horizontal line, it follows that any horizontal line drawn in the plane must intersect the graph in only one point if the function is one-to-one. We call this the **horizontal line test** for a one-to-one function, $y = f(x)$.

Example 10–4 B

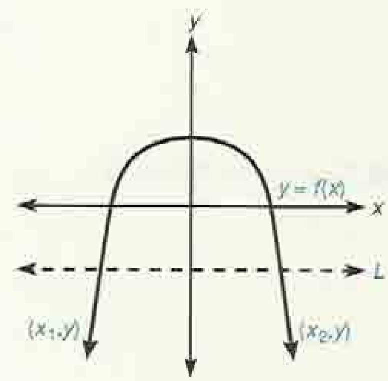
Use the horizontal line test to determine if the given graphed function is one-to-one.

1.



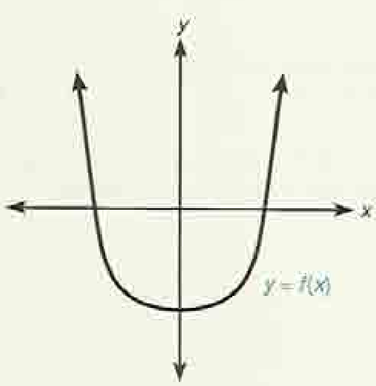
Since any horizontal line drawn in the plane will intersect the graph in only one point, the graph represents a one-to-one function.

2.



Since there exists at least one horizontal line L that can be drawn in the plane which will intersect the graph in two points, the graph does not represent a one-to-one function.

▶ ***Quick check*** Use the horizontal line test to determine if the graphed function is one-to-one.



We have seen that to obtain the inverse of a function defined by a set of ordered pairs, we interchange the components (x- and y-values) of the ordered pairs.

$$f = \{(1,2), (-2,6), (3,0), (-8,7)\}$$
$$f^{-1} = \{(2,1), (6,-2), (0,3), (7,-8)\}$$

When a one-to-one function is defined by an equation, the equation of the inverse of the function is found in the same way.

Finding the inverse of function $y = f(x)$

1. Replace $f(x)$ with y.
2. Interchange x and y in the equation.
3. Solve the resulting equation for y.
4. Replace y with $f^{-1}(x)$.

Example 10–4 C

Find the inverse of the given one-to-one function.

1. $f(x) = 4x - 3$

a. $(y) = 4x - 3$ — Replace $f(x)$ with y

b. $x = 4y - 3$ — Interchange x and y

c. Solve the equation for y.

$x + 3 = 4y$ — Add 3 to each member

$y = \dfrac{x+3}{4}$ — Divide each member by 4

d. $f^{-1}(x) = \dfrac{x+3}{4}$ — Replace y with $f^{-1}(x)$

$= \dfrac{1}{4}x + \dfrac{3}{4}$

2. $g(x) = x^3 - 3$

a. $(y) = x^3 - 3$ — Replace $g(x)$ with y

b. $x = y^3 - 3$ — Interchange x and y

c. Solve the equation for y.

$x + 3 = y^3$

$y = \sqrt[3]{x+3}$ — Take the principal cube root of each member

d. $g^{-1}(x) = \sqrt[3]{x+3}$ — Replace y with $g^{-1}(x)$

▶ ***Quick check*** Find the inverse of $f(x) = 2x + 9$

There is another important fact that we should know about inverse functions f and f^{-1}. Suppose the ordered pair (a,b) belongs to the function f. Then the ordered pair (b,a) belongs to the function f^{-1}. Consider their positions in the plane relative to the graph of the equation $y = x$. To illustrate, suppose we let $(2,4)$ be in f and then $(4,2)$ is in f^{-1}. See figure 10–2.

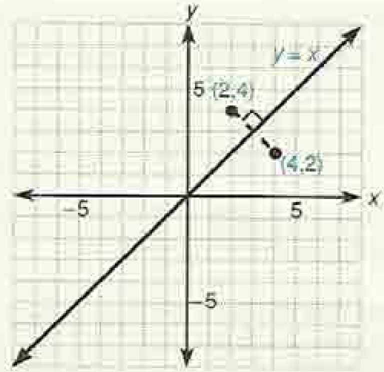


Figure 10–2

The line segment connecting points (2,4) and (4,2) is perpendicular to, and cut in half by, the line $y = x$. The points (2,4) and (4,2) are "mirror images" of each other with respect to the line $y = x$. Thus, to graph the function f^{-1}, we locate mirror images of some of the points in the graph of function f with respect to the line $y = x$.

In figure 10–3, we show the graphs of two one-to-one functions and the graphs of their inverse functions. The line segments (dashed because they are not part of either graph) join "mirror image" points in the graphs of f and f^{-1}, g, and g^{-1}.

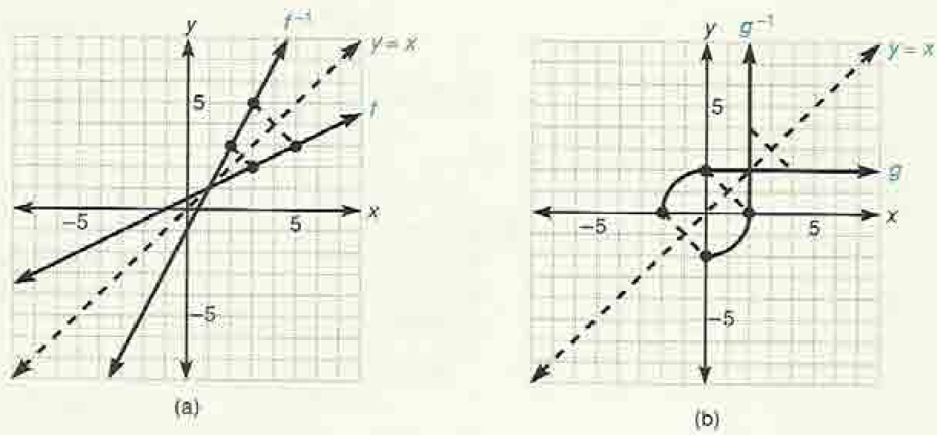


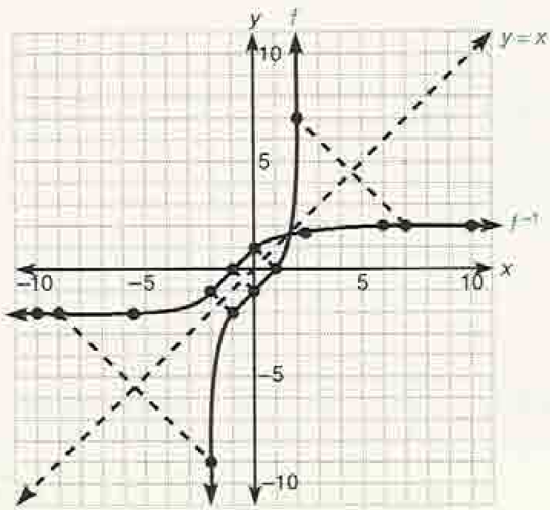
Figure 10–3

Example 10–4 D

Sketch the graph of f^{-1} on the same coordinate plane as $f(x) = x^3 - 1$. We are not familiar with the graph of $y = x^3 - 1$, so we find a number of ordered pairs and plot their graph.

x	-2	-1	0	1	2
$y = x^3 - 1$	-9	-2	-1	0	7

To determine points on the graph of f^{-1} that correspond to the points on f that we used, draw a line perpendicular to $y = x$ from the point on f. The corresponding point on f^{-1} must be at the same distance from $y = x$ on this perpendicular line as is the distance from the point on f to $y = x$.



Mastery points

Can you

- Determine if a given function is one-to-one and has an inverse function?
- Use the horizontal line test to determine if a graph represents a one-to-one function?
- Find the inverse function of a one-to-one function?
- Sketch the graph of f^{-1}, using the graph of f?

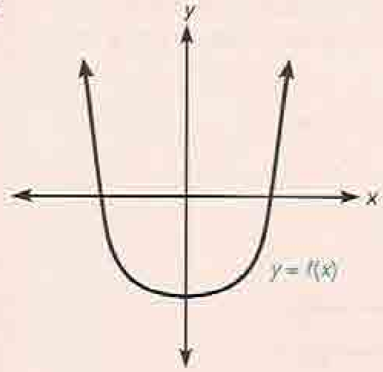
Exercise 10–4

Determine if the given functions are one-to-one functions. Use the horizontal line test on the graph where necessary. Explain the answer. See example 10–4 A.

1. $f = \{(-4,3), (2,1), (-6,-4), (0,0)\}$
2. $g = \{(-8,0), (1,2), (7,8), (3,11)\}$
3. $h = \{(1,-6), (2,3), (4,-6), (5,-7)\}$
4. $F = \{(0,-7), (3,-4), (7,6), (1,-4)\}$

See example 10–4 B.

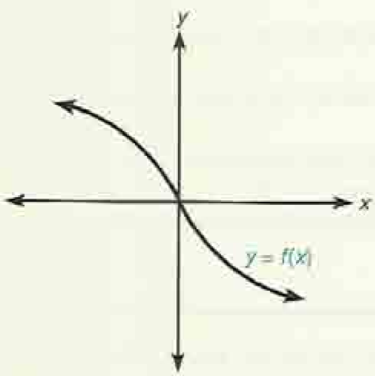
Example



Solution Since there are many horizontal lines that would intersect the graph in more than one point, the graph is one-to-one.

5. $f(x) = 2x - 7$
6. $g(x) = 8 - 3x$
7. $G(x) = x^2 - 3x + 1$
8. $H(x) = -x^2 + 2x + 4$
9. $f(x) = |x - 3|$
10. $g(x) = |2x + 4|$
11. $h(x) = \sqrt{x - 3}$
12. $F(x) = \sqrt{4 - 2x}$

13.



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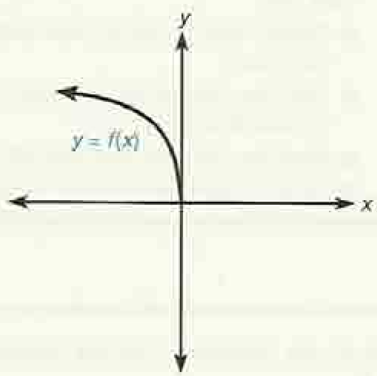

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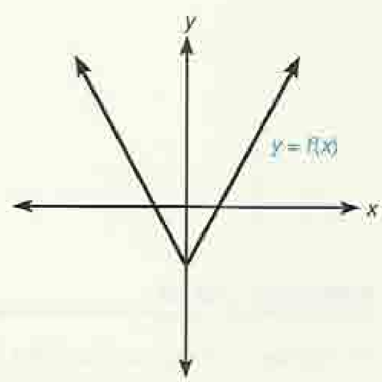

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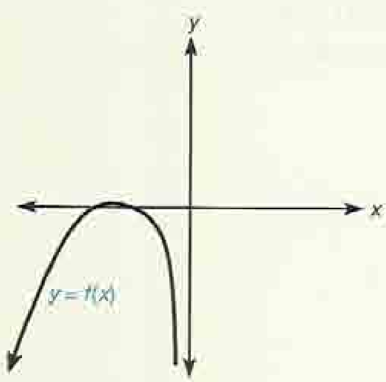

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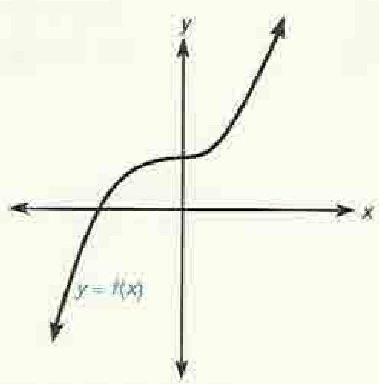

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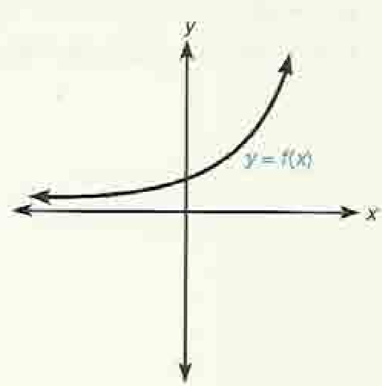


Find the inverse function of each given one-to-one function. See example 10–4 C.

Example $f(x) = 2x + 9$

Solution
1. $y = 2x + 9$ — Replace $f(x)$ with y
2. $x = 2y + 9$ — Interchange x and y
3. Solve the equation for y.

$x - 9 = 2y$ — Subtract 9 from each member

$y = \dfrac{x-9}{2}$ — Divide each member by 2

4. $f^{-1}(x) = \dfrac{x-9}{2} = \dfrac{1}{2}x - \dfrac{9}{2}$ — Replace y with $f^{-1}(x)$

19. $f = \{(-3,4), (2,-3), (0,7)\}$ **20.** $g = \{(10,-4), (-6,3), (8,2)\}$ **21.** $h(x) = 5x$

22. $F(x) = -4x$ **23.** $G(x) = 3x + 7$ **24.** $H(x) = 5x - 1$

25. $f(x) = x^3 + 2$ **26.** $g(x) = 2x^3 - 3$ **27.** $h(x) = \sqrt{x+5}, x \geq -5$

28. $F(x) = \sqrt{3x-4}, x \geq \dfrac{4}{3}$ **29.** $G(x) = \sqrt{x+2}, x \geq -2$ **30.** $H(x) = \sqrt{x} - 5, x \geq 0$

31. $f(x) = \sqrt[3]{x-2}$ **32.** $g(x) = \sqrt[3]{5x+2}$ **33.** $h(x) = \sqrt[3]{x} + 7$

34. $F(x) = \sqrt[3]{2x} - 1$

Sketch the graphs of f and f^{-1} on the same coordinate plane. See example 10–4 D.

35. $f(x) = -4x$ **36.** $f(x) = 2x$ **37.** $f(x) = x + 4$

38. $f(x) = x - 2$ **39.** $f(x) = 5x - 6$ **40.** $f(x) = 2x - 3$

41. $f(x) = -\sqrt{3-2x}, x \leq \dfrac{3}{2}$ **42.** $f(x) = \sqrt{x+2}, x \geq -2$ **43.** $f(x) = x^3 + 1$

44. $f(x) = x^3$ **45.** $f(x) = x^2 - 2, x \geq 0$ **46.** $f(x) = x^3 - 4$

47. $f(x) = -x^2 + 4, x \leq 0$

Show that $f[g(x)] = x$ and $g[f(x)] = x$ for the following functions f and g. This shows they are inverse functions.

Example $f(x) = 2x - 5$ and $g(x) = \dfrac{x+5}{2}$

Solution
1. $f[g(x)] = f\left[\dfrac{x+5}{2}\right]$ — Replace $g(x)$ with $\dfrac{x+5}{2}$

$= 2\left(\dfrac{x+5}{2}\right) - 5$ — Replace x in $f(x)$ with $\dfrac{x+5}{2}$

$= x + 5 - 5$

$= x$

2. $g[f(x)] = g(2x - 5)$ — Replace $f(x)$ with $2x - 5$

$= \dfrac{(2x-5)+5}{2}$ — Replace x in $g(x)$ with $2x - 5$

$= \dfrac{2x}{2}$

$= x$

Thus, $f[g(x)] = x = g[f(x)]$ and the functions are inverses.

48. $f(x) = 5x$ and $g(x) = \dfrac{x}{5}$ **49.** $f(x) = 4x + 7$ and $g(x) = \dfrac{x-7}{4}$

50. $f(x) = x^3 - 3$ and $g(x) = \sqrt[3]{x + 3}$

51. $f(x) = \sqrt{x - 2}$, $x \geq 2$ and $g(x) = x^2 + 2$

Review exercises

Find the solution set of the following equations. See sections 2–1 and 4–7.

1. $3y - 2 = 4$

2. $\dfrac{y}{y - 3} + \dfrac{4}{5} = \dfrac{3}{y - 3}$

3. Find the intercepts and equations of the asymptotes of the hyperbola $\dfrac{x^2}{25} - \dfrac{y^2}{9} = 1$. See section 9–4.

4. Find the equation of the line passing through $(-2,3)$ and $(4,1)$. Write the equation in standard form $ax + by = c$, $a > 0$. See section 7–3.

5. Simplify the radical expression $\sqrt{12} - \sqrt{75} + \sqrt{48}$. See section 5–5.

6. Solve the formula $S = \dfrac{n}{2}(a + \ell)$ for a. See section 2–2.

10–5 ■ Variation

If two variables are so related that as one variable changes, a change will also take place with the other variable (as with a function), we have what is called a **variation.** This relationship is the basis for many formulas that are used in the scientific and physical world. Such formulas often determine functions. For example, consider the formula for the circumference C of a circle, $C = 2\pi r$, where 2π is a constant and r is the radius of the circle. As the radius r varies, the circumference C will also vary. That is,

1. as the radius r becomes longer, the circumference C becomes longer; and
2. as the radius r becomes shorter, the circumference C becomes shorter.

This variation is an example of a **direct variation,** and we say "C varies directly as r."

Definition of direct variation

A variable y is said to *vary directly* as a variable x if

$$y = kx \quad (k > 0)$$

where k is a positive constant.

Concept

Two variables vary directly if one variable, y, is some positive multiple of the other variable, x.

In our previous example, $C = 2\pi r$, $k = 2\pi$. Another, more general, example of direct variation is where y varies directly as the nth power of x if

$$y = kx^n, \quad n > 0, \quad k > 0$$

The constant k is called the **constant of variation** (or the **constant of proportionality**).

Example 10–5 A

The formula for the area A of the surface of a sphere having radius r is given by $A = 4\pi r^2$. In this example, the constant of variation $k = 4\pi$ and "A varies directly as the square of r." An alternative terminology is "A is *directly proportional* to the second power of r."

Given the direct variation $y = kx$ and a pair of corresponding values of x and y, we can determine the value of the constant of variation k.

Example 10–5 B

Find the constant of variation k if the variable y varies directly as the variable x, and $y = 6$ when $x = 12$.

Since y varies directly as x,

$$y = kx$$
$$(6) = k \cdot (12) \quad \text{Replace } y \text{ with 6 and } x \text{ with 12}$$
$$k = \frac{6}{12} = \frac{1}{2}$$

▶ ***Quick check*** Find the constant of variation k if the variable y varies directly as the variable x and $y = 12$ when $x = 16$.

If we substitute the value for the constant of proportionality in the equation $y = kx$, we can then find the value of y given a specific value of x, and vice versa.

Example 10–5 C

1. Let y vary directly as x and suppose $y = 20$ when $x = 5$. Then find y when $x = 16$.

Since y varies directly as x,

$$y = kx$$

To find the constant of proportionality k,

$$(20) = k \cdot (5) \quad \text{Replace } y \text{ with 20 and } x \text{ with 5}$$
$$4 = k$$

Then we have the equation

$$y = (4)x \quad \text{Replace } k \text{ with 4 in } y = kx$$
$$y = 4(16) = 64 \quad \text{Replace } x \text{ with 16}$$

Therefore, $y = 64$ when $x = 16$.

2. The distance d in feet that a body falls from rest varies directly as the square of the time t in seconds (disregarding air resistance). If an object falls 144 feet in 3 seconds, how far will the object fall in 5 seconds?

Since d varies directly as the square of t,

$$d = kt^2$$
$$(144) = k(3)^2 \quad \text{Replace } d \text{ with 144 and } t \text{ with 3}$$
$$144 = k \cdot 9$$
$$k = 16$$

Then $d = (16)t^2$ Replace k with 16

$$d = 16(5)^2 \quad \text{Replace } t \text{ with 5}$$
$$= 16(25) = 400$$

The object will fall 400 feet in 5 seconds.

A second kind of variation between two variables occurs when substitution of increasing positive values of one variable results in decreasing positive values of the other variable. Such a variation is called **inverse variation.**

Definition of inverse variation

A variable y **varies inversely** as the variable x if

$$y = \frac{k}{x}, k > 0$$

and inversely as the nth power of x if

$$y = \frac{k}{x^n}, k > 0, n > 0$$

Example 10–5 D

Boyle's law for gases states that the volume V of a gas varies inversely as the pressure P, provided the mass and temperature are constant. Then

$$V = \frac{k}{P}$$

Note Alternatively, if we solve the formula for P and write the formula as $P = \frac{k}{V}$, *then* "P varies inversely as V."

Again we can determine the value of the constant of variation k given a pair of corresponding values of the variables. Using this, we can then find the value of one of the variables, given a value of the other.

Example 10–5 E

1. Given $V = \frac{k}{P}$, (a) find the constant of variation k if $V = 12$ cubic feet when $P = 100$ pounds per square foot; (b) find V when $P = 75$ pounds per square foot.

a. $V = \frac{k}{P}$

$(12) = \frac{k}{(100)}$ Replace V with 12 and P with 100

$k = 1{,}200$

b. $V = \frac{(1{,}200)}{P}$ Replace k with 1,200

$V = \frac{1{,}200}{(75)} = 16$ Replace P with 75

The volume $V = 16$ cu ft when pressure $P = 75$ lb per ft^2.

Note *Decreasing* the pressure P from 100 pounds per square foot to 75 pounds per square foot caused the volume V to *increase* from 12 cubic feet to 16 cubic feet. This is what we expect with inverse variation.

2. The resistance R, measured in ohms, of an electrical circuit in a given length of wire varies inversely as the square of the diameter of the wire measured in centimeters. If the wire has resistance $R = 0.50$ ohms when the diameter $d = 0.02$ cm, what is the resistance of the same length of wire when $d = 0.01$ cm.

Since R varies inversely as the square of d,

$$R = \frac{k}{d^2}$$

$$(0.50) = \frac{k}{(0.02)^2} \quad \text{Replace } R \text{ with } 0.50 \text{ and } d \text{ with } 0.02$$

$$0.50 = \frac{k}{0.0004}$$

$$k = 0.0002 \quad \text{Multiply each member by } 0.0004$$

$$\text{Then } R = \frac{(0.0002)}{d^2} \quad \text{Replace } k \text{ with } 0.0002$$

$$= \frac{(0.0002)}{(0.01)^2} \quad \text{Replace } d \text{ with } 0.01$$

$$= \frac{0.0002}{0.0001} = 2$$

The circuit has resistance $R = 2$ ohms when $d = 0.01$ cm.

▶ ***Quick check*** If y varies inversely as the cube of x, find y when $x = 4$ if $y = 6$ when $x = 2$.

A third variation relates one variable to two or more other variables. We call this a **joint variation.**

Definition of joint variation

A variable z is said to **vary jointly** as variables x and y if

$$z = kxy, k > 0$$

Example 10–5 F

1. The formula for the area A of a triangle with base b and altitude h is given by

$$A = \frac{1}{2}bh$$

The constant of variation is $\frac{1}{2}$ and "A varies jointly as b and h."

Alternatively, we could say that "A varies directly as b and h" or "A varies directly as the product of b and h."

2. Coulomb's law says that the magnitude F of a force between two charges of electricity is given by

$$F = k \cdot \frac{q_a \cdot q_b}{r^2}$$

where q_a and q_b are two electrical charges that are at a distance r apart. Then the force "F varies directly as the product of q_a and q_b and inversely as the square of r."

In these examples, we can determine the constant k if we are given values of the variables. We can then evaluate any one of the variables by knowing the values of the others.

■ Example 10–5 G

1. The diametral pitch P of a gear varies directly as the number of teeth N and inversely as the pitch diameter D. Find k if $P = 10$ when $N = 12$ and $D = 4$.

   If P varies directly as N and inversely as D, then

$$P = \frac{kN}{D}$$

$$(10) = \frac{k \cdot (12)}{(4)}$$ Replace P with 10, N with 12, and D with 4

$$40 = k \cdot 12$$ Multiply each member by 4

$$k = \frac{40}{12} = \frac{10}{3}$$ Divide each member by 12

2. We can now use the previous information to find N when $P = 9$ and $D = 20$.

$$(9) = \frac{\frac{10}{3}N}{(20)}$$ Replace P with 9 and D with 20

$$180 = \frac{10}{3}N$$ Multiply each member by 20

$$N = 180 \cdot \frac{3}{10} = 18 \cdot 3 = 54$$ Multiply each member by $\frac{3}{10}$

Therefore $N = 54$ when $P = 9$ and $D = 20$. ■

Mastery points

Can you

- Write an equation expressing a direct, inverse, or joint variation between variables, using the constant of variation k?
- Find the constant of variation under stated conditions?
- Find the value of one of the variables, knowing the constant k and the value(s) of the other variable(s)?

Exercise 10–5

Express the given statements as equations using the constant of variation k. See examples 10–5 A, D, and F.

1. The speed S of a falling object varies directly as the time t.
2. The perimeter P of an equilateral triangle varies directly as the side s.
3. The time t required for an automobile to travel a fixed distance is inversely proportional to the rate r.
4. The resistance R to the flow of electricity in a conductor varies inversely as the diameter d of the wire.
5. The momentum M of a body varies directly as the product of its mass m and its velocity v.
6. The current I varies directly as the product of the voltage E and the resistance R.
7. The maximum force F exerted on the vane of a wind generator varies jointly as the area A of the vane and the square of the wind velocity v.
8. The resistance R of an electrical conductor varies directly as the length ℓ and inversely as the cross-section area A.

9. The maximum torsinal stress S_{max} on a circular shaft varies directly as the torque T and inversely as the cube of the radius r.

10. The ideal gas law states that the pressure P of a gas varies directly with the absolute temperature T of the gas and inversely as the volume V of the gas.

11. The force of impact F varies directly as the product of the mass m and the velocity v and inversely as the product of the acceleration of gravity g and time t.

Find the constant of variation for the stated conditions. See examples 10–5 B and C.

Example The variable y varies directly as the variable x where $y = 12$ when $x = 16$. Find the constant of variation k.

Solution Since y varies directly as x, then

$$y = kx$$

$$(12) = k \cdot (16) \qquad \text{Replace } y \text{ with 12 and } x \text{ with 16}$$

$$k = \frac{12}{16} = \frac{3}{4} \qquad \text{Divide each member by 16}$$

12. y varies directly as x, and $x = 20$ when $y = 5$.

13. p varies directly as h, and $p = 36$ when $h = 9$.

14. A varies directly as the square of r, and $A = 154$ when $r = 7$.

15. V is directly proportional to the cube of s, and $V = 81$ when $s = 3$.

16. P varies inversely as V, and $P = 120$ when $V = 5$.

17. s varies inversely as T, and $s = 48$ when $T = 2.5$.

18. n varies inversely as the square of p, and $n = 14$ when $p = 9$.

19. p varies directly as T and inversely as V, and $p = 24$ when $T = 6$ and $V = 5$.

20. A is directly proportional to the square of b and inversely proportional to c, and $A = 42$ when $b = 3$ and $c = 3$.

21. A varies directly as the product of h and $(a + b)$, and $A = 36$ when $h = 9$, $a = 2$, and $b = 6$.

Find the value of the indicated variable. See examples 10–5 C, E, and G.

Example If y varies inversely as the cube of x, find y when $x = 4$ if $y = 6$ when $x = 2$.

Solution Since y varies inversely as the cube of x, then

$$y = \frac{k}{x^3}$$

$$(6) = \frac{k}{(2)^3} \qquad \text{Replace } y \text{ with 6 and } x \text{ with 2}$$

$$k = 6 \cdot 8 = 48$$

$$y = \frac{(48)}{x^3} \qquad \text{Replace } k \text{ with 48}$$

$$= \frac{48}{(4)^3} \qquad \text{Replace } x \text{ with 4}$$

$$= \frac{48}{64} = \frac{3}{4}$$

Thus, $y = \frac{3}{4}$ when $x = 4$.

22. x varies directly as y. If $x = 36$ when $y = 4$, find x when $y = 13$.

23. w varies directly as ℓ. If $w = 9$ when $\ell = 6$, find w when $\ell = 15$.

24. R_1 varies inversely as R_2. If $R_1 = 48$ when $R_2 = 4$, find R_1 when $R_2 = 12$.

25. If y varies inversely as the square of z, find y when $z = 3$ if $y = 12$ when $z = 2$.

26. Find A when $r = 3$ if A is directly proportional to the square of r, and $A = 48$ when $r = 4$.

$\boxed{27.}$ If v varies directly as s and inversely as t, find v when $s = 8$ and $t = 6$ if $v = 20$ when $s = 4$ and $t = 2$.

28. If T varies directly with the square of s and inversely as the cube of m, find T when $s = 2$ and $m = 3$, if $T = 14$ when $s = 3$ and $m = 6$.

Solve the following word problems.

29. The unit price P of a commodity varies inversely as the volume of sales V. If $V = 4{,}000$ units when $P = \$1.50$, find P when $V = 3{,}000$ units.

30. The velocity v of a soundwave varies directly as the product of frequency n and the wavelength ℓ. If $v = 12$ feet per second when $n = 2$ and $\ell = 4$, find v when $n = 6$ and $\ell = 8$.

31. The intensity of illumination on a surface E in foot-candles varies inversely as the square of the distance d of the light source from the surface. If $E = 6.4$ foot-candles when $d = 8$ feet, find E when $d = 6$ feet (round to the nearest tenth).

32. The field intensity H of a magnetic field varies directly as the force F acting on it and inversely as the strength of the pole m. If $H = 3$ oersteds when $F = 750$ dynes and $m = 200$, find H when $F = 500$ dynes and $m = 175$.

33. The electrical resistance R of a wire is directly proportional to the length ℓ of the wire and inversely proportional to the square of the diameter d of the wire. If $R = 8$ ohms when $\ell = 8$ feet and $d = 1$ inch, find R when $\ell = 6$ feet and $d = 2$ inches.

34. The simple interest I earned by a savings deposit in a given time t varies jointly as the principal P and the interest rate r. If $I = \$180$ when $P = \$1{,}000$ and $r = 0.06$, find I when $P = \$750$ and $r = 0.06$ in the same amount of time.

35. The maximum force F that a rectangular cantilever beam (a beam supported at one end) can withstand at its free end varies directly as the beam's width w and the square of its height h and inversely as its length ℓ. If $F = 5{,}000$ pounds when $w = 0.5$ feet, $h = 0.7$ feet, and $\ell = 12$ feet, find F when $w = 0.45$ feet, $h = 0.6$ feet, and $\ell = 18$ feet (round to the nearest tenth).

Review exercises

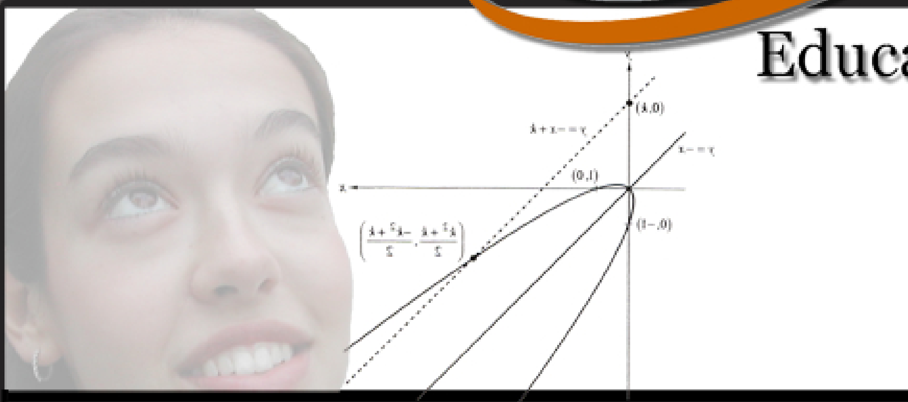
1. Find the solution set of the equation $y + 3\sqrt{y} - 10 = 0$. See section 6–6.

2. Find the equation of a line through (2,5) and perpendicular to the line $4x - y = 3$. See section 7–3.

3. Find the solution set of the inequality $|3 - x| < 5$. See section 2–6.

4. Find the value of C when $C = \dfrac{C_1C_2}{C_1 + C_2}$, $C_1 = 2$ and $C_2 = 10$. See section 1–5.

5. Simplify $\dfrac{-36a^{-2}b^4}{24a^3b^{-1}}$. Assume $a \neq 0$, $b \neq 0$. Answer with positive exponents only. See section 3–3.



Chapter 10 lead-in problem

The manufacturer of television sets has fixed costs of \$2,500,000 and an additional cost of \$350 for each set manufactured. The total cost function of manufacturing x sets is given by $C(x) = 2{,}500{,}000 + 350x$. Find the cost of manufacturing 125 sets.

Solution

Given the total cost function $C(x) = 2{,}500{,}000 + 350x$, we want $C(125)$. Now

$$\begin{aligned} C(125) &= 2{,}500{,}000 + 350(125) && \text{Replace } x \text{ with } 125 \\ &= 2{,}500{,}000 + 43{,}750 \\ &= 2{,}543{,}750 \end{aligned}$$

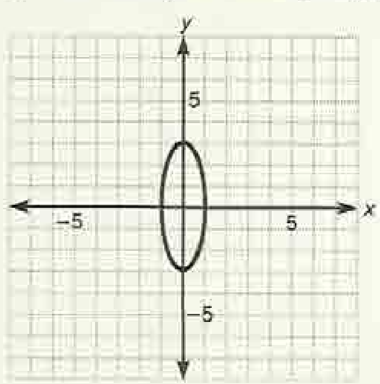
It costs \$2,543,750 to manufacture 125 sets.

Chapter 10 summary

1. A **relation** is any set of ordered pairs.
2. A **function** is a relation in which no two distinct ordered pairs have the same first components. For each x there is a unique y.
3. The **domain** of a function is the set of the first components of the ordered pairs in the function.
4. The **range** of a function is the set of the second components of the ordered pairs in the function.
5. A graph represents a function if any vertical line drawn in the plane intersects the graph in only one point.
6. A **one-to-one function** is a function in which no two ordered pairs have the same second components. For each y there is a unique x.
7. The inverse of a function f, denoted by f^{-1}, is obtained by interchanging the components in each ordered pair of the function f.
8. Given function f, f^{-1} is a function if and only if f is a one-to-one function.
9. The graphs of f and f^{-1} are mirror images (reflections) of each other with respect to the line $y = x$.
10. The composition of f and g is given by $f[g(x)]$ and the composition of g and f is given by $g[f(x)]$.
11. The following are special functions.
    a. A linear function is any function of the form $f(x) = mx + b$.
    b. A quadratic function is any function of the form $f(x) = ax^2 + bx + c$, $a \neq 0$.
    c. A constant function is any function of the form $f(x) = k$, where k is a real number.
    d. A square root function is any function of the form $f(x) = \sqrt{P}$, $P \geq 0$, where P is a polynomial in x.
12. Two variables x and y **vary directly** if $y = kx$ or $x = ky$ for some positive constant k.
13. Two variables x and y **vary inversely** if $y = \dfrac{k}{x}$ or $x = \dfrac{k}{y}$ for some positive constant k.
14. The variable z **varies jointly** as variables x and y if $z = kxy$ for some $k > 0$.

Chapter 10 error analysis

1. Range and domain of a relation
   Example: The domain of the graphed relation is $\{x|-3 \leq x \leq 3\}$ and the range is $\{y|-1 \leq y \leq 1\}$.



   Correct answer: Domain $= \{x|-1 \leq x \leq 1\}$ and range $= \{y|-3 \leq y \leq 3\}$
   What error was made? *(see page 442)*
2. Independent and dependent variable
   Example: In the equation $V = \dfrac{4}{3}\pi r^3$, the independent variable is V and the dependent variable is r.
   Correct answer: Independent r and dependent V
   What error was made? *(see page 441)*
3. Interpreting "$f(x)$"
   Example: $f(-3)$ means "f times -3."
   Correct answer: $f(-3)$ means the value of the function f at $x = -3$.
   What error was made? *(see page 449)*

4. Evaluating a function
Example: Given $f(x) = x^2 - 2x - 1$, then $f(x + h) = (x + h)^2 - 2x - 1 = x^2 + 2xh + h^2 - 2x - 1$
Correct answer:
$f(x + h) = x^2 + 2xh + h^2 - 2x - 2h - 1$
What error was made? *(see page 450)*
5. Evaluating the composition of functions
Example: Given $f(x) = x^2 + 2$ and $g(x) = 4x - 3$, then $f[g(-2)] = f[(-2)^2 + 2] = f(6) = 4(6) - 3 = 21$
Correct answer: $f[g(-2)] = 123$
What error was made? *(see page 451)*
6. Domain of a square root function
Example: Given $f(x) = \sqrt{x + 3}$, the domain $= \{x|-3 \leq x \leq 3\}$.
Correct answer: Domain $= \{x|x \geq -3\}$
What error was made? *(see page 458)*
7. Inverse functions
Example: The symbol $f^{-1} = \dfrac{1}{f}$.
Correct answer: f^{-1} is the symbol for the inverse function of function f.
What error was made? *(see page 461)*
8. One-to-one functions
Example: The function $f(x) = 3x - 1$ is a one-to-one function since for each value of x there is a unique (one and only one) value of y.
Correct answer: f is one-to-one because for every y there is a unique (one and only one) value of x.
What error was made? *(see page 462)*
9. Inverse of a function
Example: Given $f(x) = x^3 - 7$, then $y = x^3 - 7$, $y^3 = x - 7$, $y = \sqrt[3]{x - 7}$, $f^{-1}(x) = \sqrt[3]{x - 7}$
Correct answer: $f^{-1}(x) = \sqrt[3]{x + 7}$
What error was made? *(see page 463)*
10. Variation
Example: Given y is directly proportional to the square of x and inversely proportional to the cube of z, then $y = \dfrac{kz^3}{x^2}$.
Correct answer: $y = \dfrac{kx^2}{z^3}$
What error was made? *(see page 472)*

Chapter 10 critical thinking

Choose a three-digit number greater than 100 where all three digits are the same (such as 444 or 777). Add the three digits and divide the original number by this sum. No matter what three-digit number you choose, the answer is 37. Why is this true?

Chapter 10 review

[10–1]

Determine whether or not each relation defines a function. If not, explain why not.

1. $\{(1,3), (2,-4), (1,0)\}$
2. $\{(4,3), (-2,3), (1,3)\}$
3. $\{(0,7), (-7,1), (6,-3)\}$
4. $2x + y = 3$
5. $\{(x,y)|y = 5\}$
6. $\{(x,y)|x = -7\}$

Determine the domain and range of each relation.

7. $\{(-3,4), (4,2), (0,0), (-2,7)\}$
8. $\{(x,y)|y = -4x + 1\}; x \in \{-3,-2,-1,0,1,2,3\}$
9. $\{(x,y)|y = x^2 + 2x - 1\}; x \in \{-3,0,1,4\}$
10. $\{(x,y)|y = -7\}; x \in \{-8,-2,0,3,7\}$

Determine the domain of each relation.

11. $\{(x,y)|y = 2x + 7\}$
12. $\{(x,y)|y = 2x^2 + 9\}$
13. $\left\{(x,y)|y = \dfrac{-3}{x}\right\}$
14. $\left\{(x,y)|y = \dfrac{3}{3x - 4}\right\}$
15. $\{(x,y)|y = \sqrt{x + 2}\}$

[10–2]

Given $f(x) = x + 2$ and $g(x) = x^2 - 3$, find each of the following.

16. $f(-2)$ **17.** $f(-4)$ **18.** $g(0)$ **19.** $f(3) - g(2)$

20. $f[g(-3)]$ **21.** $g[f(0)]$ **22.** $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$

[10–3]

Identify each function by name and sketch the graph.

23. $f(x) = 3x + 5$ **24.** $f(x) = -7$ **25.** $f(x) = x^2 - 6x + 5$

26. $f(x) = \sqrt{x + 5}$

[10–4]

In each of the following, determine if the given function is one-to-one. Use the horizontal line test if necessary.

27. $f = \{(-3,1), (4,3), (2,1)\}$ **28.** $g = \{(0,2), (4,-3), (6,3)\}$ **29.** $f(x) = 5 - 4x$

30. $g(x) = x^2 + 1$ **31.** $h(x) = |2x - 3|$ **32.** $F(x) = \sqrt{x + 4}$

Find the inverse function, f^{-1}, of each given function.

33. $f(x) = 4x + 3$ **34.** $f(x) = x^2 - 2$, $x \geq 0$ **35.** $f(x) = \sqrt{2x - 3}$, $x \geq \dfrac{3}{2}$

Sketch the graph of each function f and its inverse f^{-1} on the same set of coordinate axes.

36. $f(x) = 3x - 2$ **37.** $f(x) = \sqrt{4 - x}$, $x \leq 4$ **38.** $f(x) = x^2 - 7$, $x \geq 0$

[10–5]

In problems 39–42, express each statement as an equation using a constant of variation k.

39. x varies directly as the square of y.

40. The current I is inversely proportional to the resistance R.

41. The angular momentum H of a particle varies jointly as the linear velocity v and the radius of rotation r.

42. The resistance R of a conductor varies directly as the length ℓ and inversely as the area A.

43. If s varies directly as w^3, find the constant of variation k if $s = 16$ when $w = 2$.

44. If y varies inversely as the square of x, find k if $y = 24$ when $x = 3$.

45. If R varies directly as ℓ and inversely as A, find k if $R = 0.45$ when $\ell = 5.0$ and $A = 0.055$.

46. P varies jointly as R and I^2. If $P = 1{,}500$ when $R = 20$ and $I = 5$, find P when $R = 20$ and $I = 4$.

47. d varies jointly as a and b and inversely as c. If $d = 15$ when $a = 3$, $b = 7$, and $c = 14$, find d when $a = 5$, $b = 8$, and $c = 20$.

Chapter 10 cumulative test

[1–4] **1.** Perform the indicated operations and simplify.
$$\frac{4(2+5) - 4^2 + 9}{(-2)(-7)}$$

[1–5] **2.** Given $P(x) = x^3 - 4x + 1$, find (a) $P(1)$, (b) $P(-3)$, (c) $P(0)$.

[1–5] **3.** Given $S = V_0 t + \dfrac{1}{2}at^2$, find S when $V_0 = 20$, $t = 3$, $a = 32$.

Perform the indicated operations. Assume all variable exponents are positive integers. Assume all denominators are nonzero. Express answers with positive exponents only.

[3–1] 4. $(x^4y^5)^2(-x^5y^8)$ [3–3] 5. $(2x-1)^0$ [3–3] 6. $\dfrac{x^{-4}y^{-5}}{x^2y^2}$

[3–3] 7. 4^{-2} [3–3] 8. $\left(\dfrac{1}{2}\right)^{-3}$ [3–3] 9. $\dfrac{4^{-6}}{4^{-4}}$

Completely factor the following expressions.

[3–7] 10. $3a^2-27b^2$ [3–7] 11. y^4-x^4 [3–4] 12. $5x(2x+3)-10(2x+3)$

[3–6] 13. $6a^2+7a-3$ [3–6] 14. $2a^2b^2-ab-1$ [3–7] 15. $2x^3-54y^3$

[2–2] 16. Solve $ax-by=2x+1$ for x.

Find the solution set of the following equations and inequalities.

[2–1] 17. $-3(a+5)+4(2a-1)=4$ [2–4] 18. $-3\le 2x+1<5$

[6–7] 19. $x^2-6x-16\ge 0$ [4–7] 20. $\dfrac{3}{y}+\dfrac{2}{y}=\dfrac{7}{3}$

[6–1] 21. $y-\dfrac{3}{y}=2$

Perform the indicated operations. Assume all variables are nonzero. Answer with positive exponents only.

[5–1] 22. $(a^{-3/4})^{1/2}$ [5–1] 23. $\dfrac{b^{5/4}}{b}$ [5–1] 24. $(x^4y^6)^{1/2}$

Find the solution set of the following systems of linear equations.

[8–1] 25. $x-2y=6$
$3x+y=-1$

[8–6] 26. $3y-2x=0$
$2y+4x=3$ Use an augmented matrix.

Given $f(x)=5-6x$ and $g(x)=x^2+x-1$, find

[10–2] 27. $f(-3)$ [10–2] 28. $f[g(2)]$ [10–2] 29. $g[f(-1)]$

[10–4] 30. Given $f(x)=4x+3$, find $f^{-1}(x)$.

[10–4] 31. Given $f(x)=x^3+1$ and $g(x)=\sqrt[3]{x-1}$, show that f and g are inverse functions by verifying that (a) $f[g(x)]=x$, (b) $g[f(x)]=x$.

Find the x- and y-intercepts of the graphs of the following equations, if they exist. What type of figure is each?

[9–1] 32. $y=x^2+5x-40$ [9–3] 33. $x^2+2y^2=8$

[9–3] 34. $\dfrac{x^2}{16}-\dfrac{y^2}{25}=1$ [9–1] 35. $y=-x^2-8x+20$

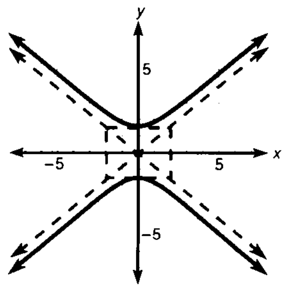
Give a name to each function. Sketch its graph.

[10–3] 36. $f(x)=7$ [10–3] 37. $f(x)=4x-6$ [10–3] 38. $f(x)=x^2-4x-5$

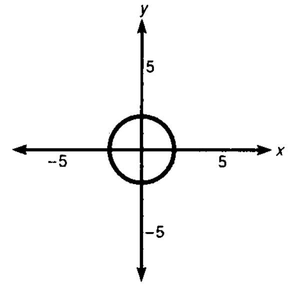
[10–3] 39. $f(x)=\sqrt{x+9}$, $x\ge -9$ [10–3] 40. $f(x)=|x|-3$

[10–5] 41. Given the variable y varies directly as the variable x and inversely as the cube of the variable z, (a) find the constant of variation k if $y=16$ when $x=4$ and $z=2$, (b) find z when $y=40$ and $x=5$.

38. hyperbola, $\frac{y^2}{2} - \frac{x^2}{4} = 1$



39. circle, $x^2 + y^2 = 4$



40. $\{(-4,16), (2,4)\}$
41. $\{(-2,-\sqrt{3}), (-2,\sqrt{3}), (2,-\sqrt{3}), (2,\sqrt{3})\}$

Chapter 10

Exercise 10–1

Answers to odd-numbered problems

1. domain $= \{8,5,9,6\}$, range $= \{0,3,4\}$ **3.** domain $= \{-4,1\}$, range $= \{1,2,3,9\}$ **5.** domain $= \{6,1,2,3\}$, range $= \{-1,1\}$ **7.** domain $= \{5,6,7\}$, range $= \{3,-4\}$ **9.** domain $= \{x \mid -2 \le x \le 2\}$, range $= \{y \mid -2 \le y \le 2\}$ **11.** domain $= \{x \mid x \in R\}$, range $= \{y \mid y \le 3\}$ **13.** domain $= \{x \mid x \in R\}$, range $= \{y \mid y \in R\}$ **15.** domain $= \{x \mid x \le -2 \text{ or } x \ge 2\}$, range $= \{y \mid y \in R\}$ **17.** function **19.** not a function because two of the ordered pairs have the same first component (2,3) and (2,−7) **21.** not a function because two of the ordered pairs have the same first component (−1,2) and (−1,6) **23.** function **25.** function **27.** function **29.** not a function because two of the ordered pairs have the same first component (4,−2) and (4,2) **31.** function **33.** not a function since two ordered pairs have the same first component, (−10,0) and (−10,1) **35.** not a function since two ordered pairs have the same first component, (3,1) and (3,4) **37.** domain $= \{-3,-1,0,1,3\}$ **39.** domain $= \{-5,-3,0,3,5\}$ **41.** domain $= \{-5,-1,1,2,4\}$ **43.** domain $= \{x \mid x \in R\}$ **45.** domain $= \{x \mid x \in R\}$ **47.** domain $= \{x \mid x \in R, x \neq 0\}$ **49.** domain $= \left\{x \mid x \in R, x \neq -\frac{3}{2}\right\}$ **51.** domain $= \left\{x \mid x \ge -\frac{4}{3}\right\}$ **53.** function since any vertical line intersects the graph at only one point **55.** function since any vertical line intersects the graph at only one point **57.** not a function since at least one vertical line intersects at more than one point on the graph **59.** not a function since at least one vertical line intersects at more than one point on the graph

Solutions to trial exercise problems

5. domain is the set of first components $= \{6,1,2,3\}$, range is the set of second components $= \{-1,1\}$ **9.** domain is set of all values of $x = \{x \mid -2 \le x \le 2\}$, range is the set of all values of $y = \{y \mid -2 \le y \le 2\}$ **22.** does define a function since no two ordered pairs have the same first component **31.** defines a function—no two ordered pairs have the same first component **46.** domain $= \{x \mid x \in R, x \neq 0\}$, since $xy = 2$ becomes $y = \frac{2}{x}$ if we solve for y.

Review exercises

1. $\frac{4x^8}{y^6}$ **2.** x^6 **3.** y^4 **4.** $x^{y+1} \neq x^y \cdot y$ since the bases, x and y, are not the same **5.** $2(x + 2y)(x - 2y)$ **6.** $3\sqrt{6}$ units **7.** 60 m by 120 m

Exercise 10–2

Answers to odd-numbered problems

1. $f(0) = -2$; $(0,-2)$ **3.** $f\left(\frac{2}{3}\right) = 0$; $\left(\frac{2}{3},0\right)$ **5.** $g(7) = 58$; $(7,58)$ **7.** $g\left(\frac{1}{2}\right) = -\frac{15}{4}$; $\left(\frac{1}{2},-\frac{15}{4}\right)$ **9.** $f(a + 1) = 3a + 1$; $(a + 1, 3a + 1)$ **11.** $f(a^2) = 3a^2 - 2$; $(a^2, 3a^2 - 2)$ **13.** 27 **15.** 16 **17. a.** $4x^2 + 8hx + 4h^2$ **b.** $8hx + 4h^2$ **c.** $8x + 4h$ **19. a.** $2x^2 + 4hx + 2h^2 + 3x + 3h + 2$ **b.** $4hx + 2h^2 + 3h$ **c.** $4x + 2h + 3$ **21. a.** $5 - 2x - 2h$ **b.** $-2h$ **c.** -2 **23.** $f(-5) = -17$, $(-5,-17)$; $f(0) = -2$, $(0,-2)$; $f\left(\frac{2}{3}\right) = 0$, $\left(\frac{2}{3},0\right)$ **25.** $h\left(-\frac{1}{2}\right) = \frac{11}{4}$, $\left(-\frac{1}{2},\frac{11}{4}\right)$; $h(0) = 1$, $(0,1)$; $h(3) = 22$, $(3,22)$ **27.** $g(-15) = 10$, $(-15,10)$; $g(0) = 10$, $(0,10)$; $g\left(\frac{6}{5}\right) = 10$, $\left(\frac{6}{5},10\right)$ **29.** $48x^2 + 24x - 2$ **31.** $27x^4 - 90x^2 + 70$ **33.** 7 **35.** −6 **37.** (14,−10), (32,0), (212,100) **39.** (1,1), (3,27), (5,125); domain $= \{s \mid s > 0\}$ **41.** (2,49), (3,69), (5,109)

Solutions to trial exercise problems

9. $f(a + 1) = 3(a + 1) - 2 = 3a + 3 - 2 = 3a + 1$; $(a + 1, 3a + 1)$ **12.** $f(5) = 3(5) - 2 = 15 - 2 = 13$ and $f(2) = 3(2) - 2 = 4$. So $f(5) - f(2) = 13 - 4 = 9$.

19. a.
$$\begin{aligned} f(x + h) &= 2(x + h)^2 + 3(x + h) + 2 \\ &= 2(x^2 + 2xh + h^2) + 3x + 3h + 2 \\ &= 2x^2 + 4xh + 2h^2 + 3x + 3h + 2 \end{aligned}$$

b.
$$\begin{aligned} f(x + h) - f(x) &= (2x^2 + 4xh + 2h^2 + 3x + 3h + 2) \\ &\quad - (2x^2 + 3x + 2) \\ &= 2x^2 + 4xh + 2h^2 + 3x + 3h + 2 - 2x^2 \\ &\quad - 3x - 2 \\ &= 4xh + 2h^2 + 3h \end{aligned}$$

c.
$$\begin{aligned} \frac{f(x + h) - f(x)}{h} &= \frac{4xh + 2h^2 + 3h}{h} = \frac{h(4x + 2h + 3)}{h} \\ &= 4x + 2h + 3 \text{ since } h \neq 0 \end{aligned}$$

23. $f(x) = 3x - 2$

$f(-5) = 3(-5) - 2 = -17; (-5,-17)$

$f(0) = 3(0) - 2 = -2; (0,-2)$

$f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right) - 2 = 0; \left(\frac{2}{3},0\right)$

26. $g(x) = -7$

$g\left(-\frac{7}{8}\right) = -7; \left(-\frac{7}{8},-7\right)$

$g(0) = -7; (0,-7)$

$g(25) = -7; (25,-7)$

29. $f[g(x)] = f[4x + 1]$

$= 3(4x + 1)^2 - 5$

$= 3(16x^2 + 8x + 1) - 5$

$= 48x^2 + 24x + 3 - 5$

$= 48x^2 + 24x - 2$

34. $f[g(4)] = f[4(4) + 2] = f(18) = 3(18)^2 - 5 = 967$

37. $g(F) = \frac{5}{9}(F - 32)$

$g(14) = \frac{5}{9}(14 - 32) = \frac{5}{9}(-18) = -10; (14,-10)$

$g(32) = \frac{5}{9}(32 - 32) = \frac{5}{9}(0) = 0; (32,0)$

$g(212) = \frac{5}{9}(212 - 32) = \frac{5}{9}(180) = 100; (212,100)$

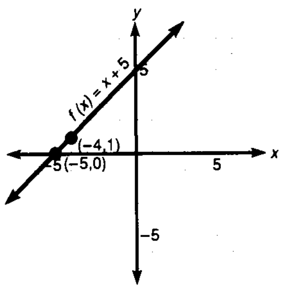
Review exercises

1. $6x^2 + 7x - 14$ **2.** $-x^2 - 3x + 6$
3. x-intercepts, $(2 - 2\sqrt{2},0)$, $(2 + 2\sqrt{2},0)$; y-intercept, $(0,-4)$; vertex, $(2,-8)$ **4.** $m = \frac{4}{5}$; y-intercept, $(0,4)$
5. fourth degree **6.** $3x^2 - x + 1 + \frac{4}{x - 1}$

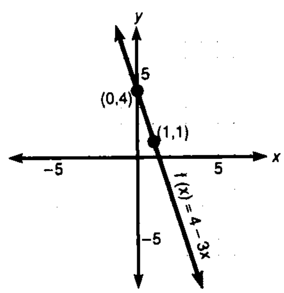
Exercise 10–3

Answers to odd-numbered problems

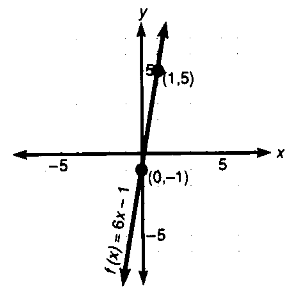
1. linear



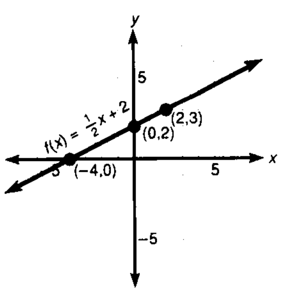
3. linear



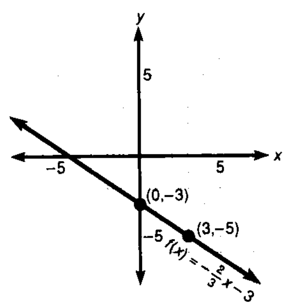
5. linear



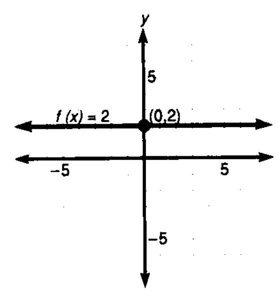
7. linear



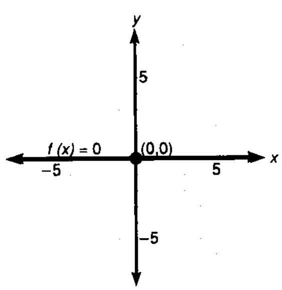
9. linear



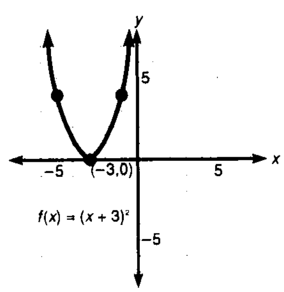
11. constant, linear



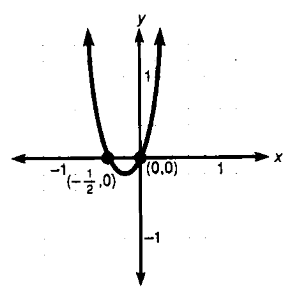
13. constant, linear



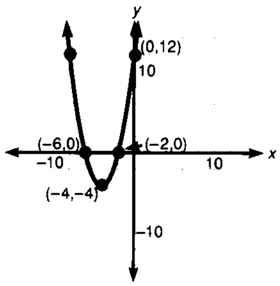
15. quadratic



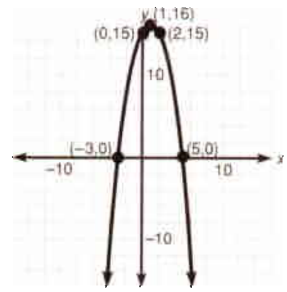
17. quadratic



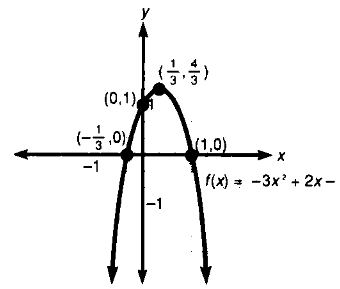
19. quadratic



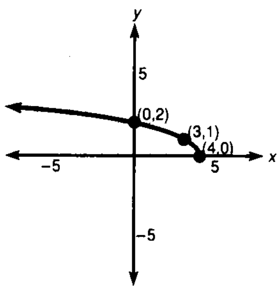
21. quadratic



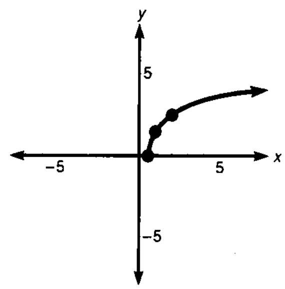
23. quadratic



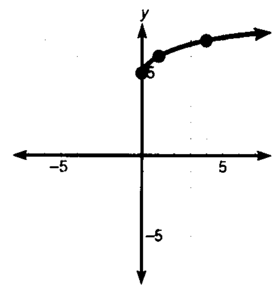
25. square root



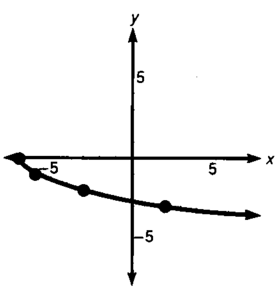
27. square root



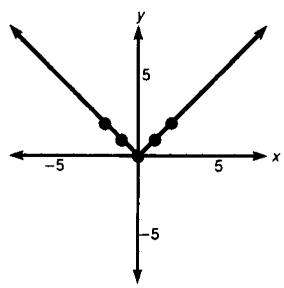
29. square root



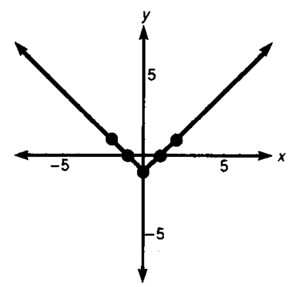
31. square root



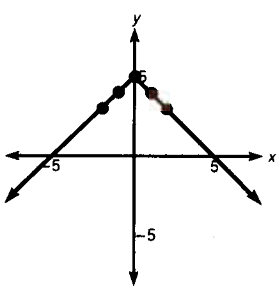
33. $f(-2) = 2, f(-1) = 1, f(0) = 0,$
$f(1) = 1, f(2) = 2$



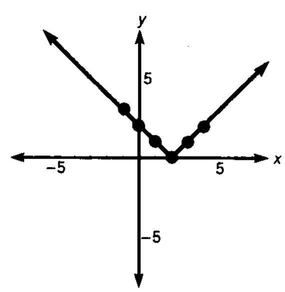
35. $f(-2) = 1, f(-1) = 0, f(0) = -1,$
$f(1) = 0, f(2) = 1$



37. $f(-2) = 3, f(-1) = 4, f(0) = 5,$
$f(1) = 4, f(2) = 3$

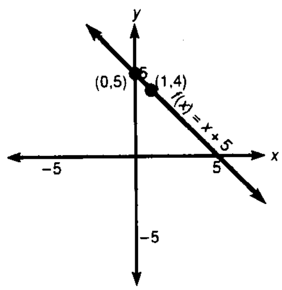


39. $f(-1) = 3, f(0) = 2, f(1) = 1,$
$f(2) = 0, f(3) = 1, f(4) = 2$

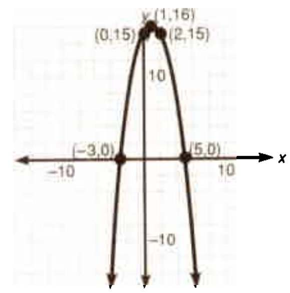


Solutions to trial exercise problems

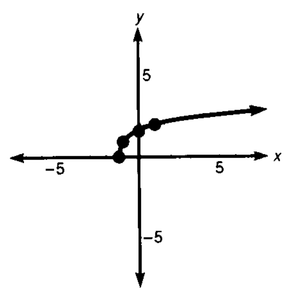
4. $f(x) = 5 - x$ is a linear function since the largest power of the variable is one. Use y-intercept 5 and slope $m = -1$.



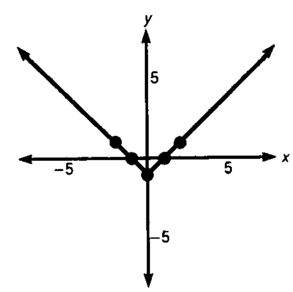
21. $f(x) = -x^2 + 2x + 15$ is a quadratic function whose graph is a parabola opening downward. The y-intercept is 15, x-intercepts are 5 and -3, and the vertex is $(1,16)$.



28. $f(x) = \sqrt{2x + 3}$ is a square root function with x-intercept $-\dfrac{3}{2}$, y-intercept $\sqrt{3} \approx 1.7$ and is part of a parabola (above the x-axis) opening to the right.



35. $f(x) = |x| - 1$ is an absolute value function with y-intercept -1, x-intercepts $1, -1$ and is "V" shaped.



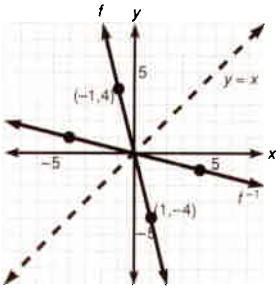
Review exercises

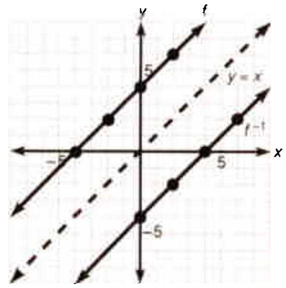
1. $\dfrac{7(5a - b)}{4}$ **2.** $\left\{\left(-\dfrac{2}{11}, -\dfrac{7}{11}\right)\right\}$ **3.** $\{z \mid z \leq -1 \text{ or } z \geq 3\} = (-\infty, -1] \cup [3, \infty)$ **4.** $\dfrac{3}{4}$ **5.** $\dfrac{4\sqrt{5}}{5}$ **6.** $30 - 12\sqrt{6}$ **7.** 25

Exercise 10–4

Answers to odd-numbered problems

1. one-to-one function since no two ordered pairs have the same second component **3.** not a one-to-one function since $(1,-6)$ and $(4,-6)$ have the same second components **5.** one-to-one function since it is a linear function where $m \neq 0$ **7.** not a one-to-one function since $G(1) = -1$ and $G(2) = -1$ **9.** not a one-to-one function since at least two of the ordered pairs have the same second component $(1,2)$ and $(5,2)$ **11.** one-to-one function since $h(x)$ has a different value for every $x \geq 3$ **13.** one-to-one function since any horizontal line drawn in the plane intersects the graph at only one point **15.** not a one-to-one since a horizontal line intersects the graph in more than one point **17.** one-to-one function since any horizontal line does not intersect the graph in more than one point **19.** $f^{-1}(x) = \{(4,-3), (-3, 2), (7,0)\}$ **21.** $h^{-1}(x) = \dfrac{x}{5}$ **23.** $G^{-1}(x) = \dfrac{x - 7}{3}$ **25.** $f^{-1}(x) = \sqrt[3]{x - 2}$ **27.** $h^{-1}(x) = x^2 - 5, x \geq 0$ **29.** $G^{-1}(x) = x^2 - 2, x \geq 0$ **31.** $f^{-1}(x) = x^3 + 2$ **33.** $h^{-1}(x) = (x - 7)^3$

35.


37.


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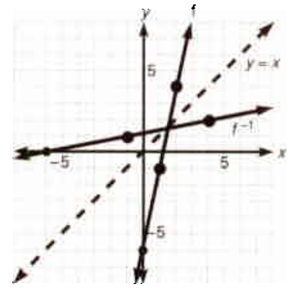

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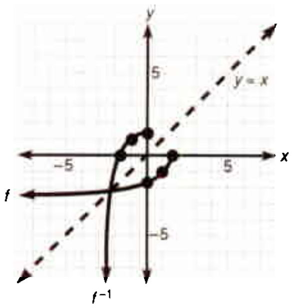
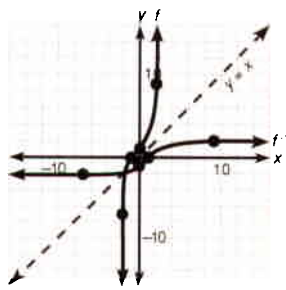
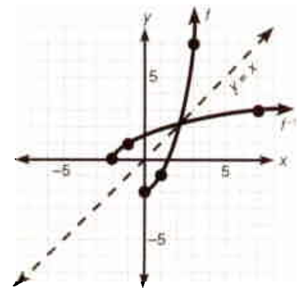
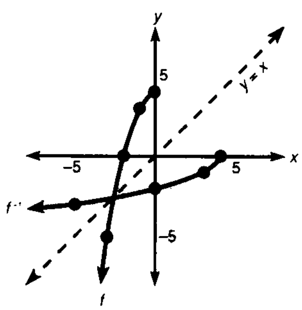


43.


45.


47.


49. a. $f[g(x)] = f\left(\frac{x-7}{4}\right) = 4\left(\frac{x-7}{4}\right) + 7 = x - 7 + 7 = x$

b. $g[f(x)] = g(4x + 7) = \frac{(4x + 7) - 7}{4} = \frac{4x}{4} = x$

51. a. $f[g(x)] = f(x^2 + 2) = \sqrt{(x^2 + 2) - 2} = \sqrt{x^2} = x$

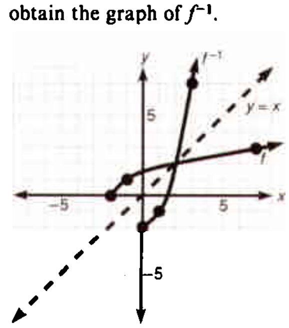
b. $g[f(x)] = g(\sqrt{x - 2}) = (\sqrt{x - 2})^2 + 2 = x - 2 + 2 = x$

Solutions to trial exercise problems

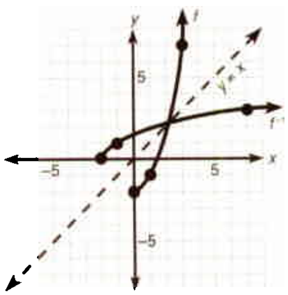
7. G is not a one-to-one function since the graph of G is a parabola opening up and there exists a horizontal line that would intersect the graph in two points. 11. h is one-to-one since no two ordered pairs have the same second component, for $x \geq 3$.

25. Given $f(x) = x^3 + 2$, then $y = x^3 + 2$ and interchanging variables $x = y^3 + 2$, then $y^3 = x - 2$ and $y = \sqrt[3]{x - 2}$. Thus $f^{-1}(x) = \sqrt[3]{x - 2}$.

42. Graph the function $f(x) = \sqrt{x + 2}$. Choose points on this curve for $x \geq -2$ and locate other points on the other side of the line $y = x$, the same distance away from this line. Connect these points to obtain the graph of f^{-1}.



45. Graph the function $f(x) = x^2 - 2$ for $x \geq 0$. Choose points on this curve that are on the other side of the line $y = x$, the same distance from this line. Connect these points to obtain the graph of f^{-1}.



Review exercises

1. $\{2\}$ **2.** $\emptyset$; 3 is extraneous **3.** x-intercepts, $(-5,0)$, $(5,0)$; no y-intercepts, $y = -\frac{3}{5}x$ and $y = \frac{3}{5}x$ **4.** $x + 3y = 7$ **5.** $\sqrt{3}$ **6.** $a = \dfrac{2S - n\ell}{n}$

Exercise 10-5

Answers to odd-numbered problems

1. $S = kt$ **3.** $t = \dfrac{k}{r}$ **5.** $M = kmv$ **7.** $F = kAv^2$ **9.** $S_{max} = \dfrac{kT}{r^3}$ **11.** $F = \dfrac{kmv}{gt}$ **13.** $p = kh$, $k = 4$ **15.** $V = ks^3$, $k = 3$ **17.** $s = \dfrac{k}{T}$, $k = 120$ **19.** $p = \dfrac{kT}{V}$, $k = 20$ **21.** $A = kh(a + b)$, $k = \dfrac{1}{2}$ **23.** $w = k\ell$, $k = \dfrac{3}{2}$, $w = \dfrac{45}{2}$ **25.** $y = \dfrac{k}{z^2}$, $k = 48$, $y = \dfrac{16}{3}$ **27.** $v = \dfrac{ks}{t}$, $k = 10$, $v = \dfrac{40}{3}$ **29.** $V = \dfrac{k}{P}$; $k = 6{,}000$; $P = \$2.00$ **31.** $E = \dfrac{k}{d^2}$, $k = 409.6$, $E = 11.4$ footcandles **33.** $R = \dfrac{k\ell}{d^2}$, $k = 1$, $R = \dfrac{3}{2}$ ohms **35.** $F = \dfrac{kwh^2}{\ell}$, $k = 244{,}898.0$, $F = 2{,}204.1$ lb

Solutions to trial exercise problems

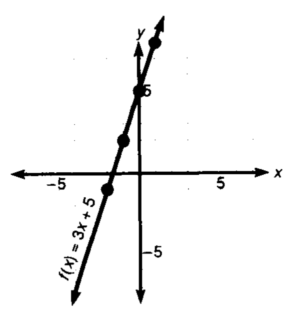
5. Since M varies directly as the product of m and v, $M = kmv$.
8. Since R varies directly as ℓ and inversely as A, $R = \dfrac{k\ell}{A}$.
18. $n = \dfrac{k}{p^2}$, $n = 14$ when $p = 9$, so $14 = \dfrac{k}{(9)^2} = \dfrac{k}{81}$, $k = 14 \cdot 81 = 1{,}134$. **27.** $v = \dfrac{ks}{t}$. $v = 20$ when $s = 4$ and $t = 2$, so $20 = \dfrac{k \cdot 4}{2}$ and $k = 10$. Then $v = \dfrac{10 \cdot s}{t}$ and $s = 8$ when $t = 6$, so $v = \dfrac{10 \cdot 8}{6} = \dfrac{40}{3}$.

Review exercises

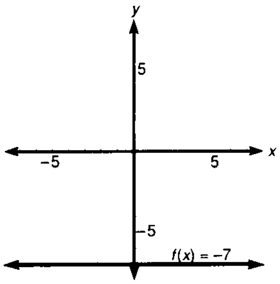
1. $\{4\}$; 25 is extraneous **2.** $x + 4y = 22$ **3.** $\{x|-2 < x < 8\} = (-2,8)$ **4.** $C = \dfrac{5}{3}$ **5.** $\dfrac{-3b^5}{2a^5}$

Chapter 10 review

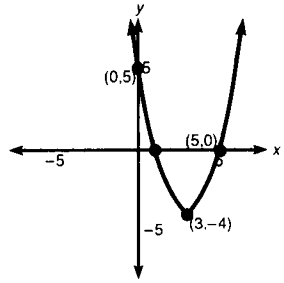
1. not a function since two of the ordered pairs have the same first component $(1,3)$ and $(1,0)$ **2.** function **3.** function **4.** function **5.** function **6.** not a function since $x = k$; does *not* define a function; (vertical line) **7.** domain $= \{-3,4,0,-2\}$, range $= \{4,2,0,7\}$ **8.** domain $= \{-3,-2,-1,0,1,2,3\}$, range $= \{13,9,5,1,-3,-7,-11\}$ **9.** domain $= \{-3,0,1,4\}$, range $= \{2,-1,23\}$ **10.** domain $= \{-8,-2,0,3,7\}$, range $= \{-7\}$ **11.** domain $= \{x|x \in R\}$ **12.** domain $= \{x|x \in R\}$ **13.** domain $= \{x|x \in R, x \neq 0\}$ **14.** domain $= \left\{x|x \in R, x \neq \frac{4}{3}\right\}$ **15.** domain $= \{x|x \geq -2\}$ **16.** 0 **17.** -2 **18.** -3 **19.** 4 **20.** 8 **21.** 1 **22.** 1 **23.** linear



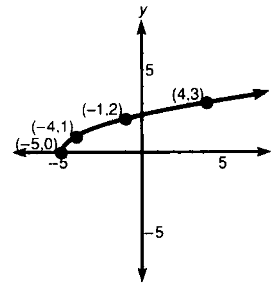
24. constant, linear



25. quadratic



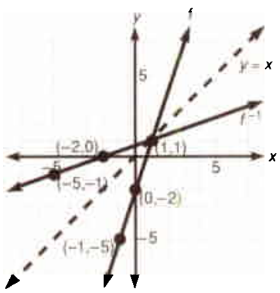
26. square root



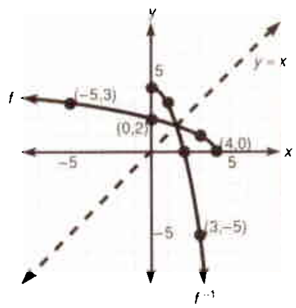
27. not one-to-one function because two ordered pairs have the same second component **28.** one-to-one function **29.** one-to-one function **30.** not one-to-one function because the horizontal line test fails **31.** not one-to-one function because the horizontal line test fails **32.** one-to-one function **33.** $f^{-1}(x) = \dfrac{x-3}{4}$

34. $f^{-1}(x) = \sqrt{x+2}, x \geq -2$ **35.** $f^{-1}(x) = \dfrac{1}{2}x^2 + \dfrac{3}{2}, x \geq 0$

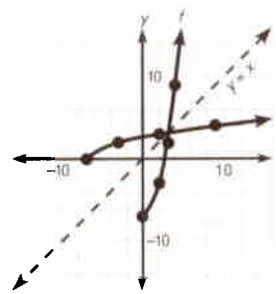
36.



37.



38.



39. $x = ky^2$ **40.** $I = \dfrac{k}{R}$ **41.** $H = kvr$ **42.** $R = \dfrac{k\ell}{A}$

43. $s = kw^3, k = 2$ **44.** $y = \dfrac{k}{x^2}, k = 216$ **45.** $R = \dfrac{k\ell}{A}$, $k = .00495$ **46.** $P = kRI^2, k = 3, P = 960$ **47.** $d = \dfrac{kab}{c}$, $k = 10, d = 20$

Chapter 10 cumulative test

1. $\dfrac{3}{2}$ **2. a.** -2 **b.** -14 **c.** 1 **3.** 204 **4.** $-x^{13}y^{18}$ **5.** 1

6. $\dfrac{1}{x^6y^7}$ **7.** $\dfrac{1}{16}$ **8.** 8 **9.** $\dfrac{1}{16}$ **10.** $3(a+3b)(a-3b)$

11. $(x^2+y^2)(x+y)(y-x)$ **12.** $5(x-2)(2x+3)$

13. $(3a-1)(2a+3)$ **14.** $(ab-1)(2ab+1)$

15. $2(x-3y)(x^2+3xy+9y^2)$ **16.** $x = \dfrac{1+by}{a-2}$ **17.** $\left\{\dfrac{23}{5}\right\}$

18. $\{x \mid -2 \leq x < 2\}$ **19.** $\{x \mid x \leq -2 \text{ or } x \geq 8\}$ **20.** $\left\{\dfrac{15}{7}\right\}$

21. $\{3,-1\}$ **22.** $\dfrac{1}{a^{3/8}}$ **23.** $b^{1/4}$ **24.** x^2y^3 **25.** $\left\{\left(\dfrac{4}{7}, -\dfrac{19}{7}\right)\right\}$

26. $\left\{\left(\dfrac{9}{16}, \dfrac{3}{8}\right)\right\}$ **27.** 23 **28.** -25 **29.** 131

30. $f^{-1}(x) = \dfrac{x-3}{4}$ **31. a.** $f[g(x)] = f[\sqrt[3]{x-1}]$ $= (\sqrt[3]{x-1})^3 + 1 = x - 1 + 1 = x$
b. $g[f(x)] = g[x^3+1] = \sqrt[3]{(x^3+1)-1} = \sqrt[3]{x^3} = x$

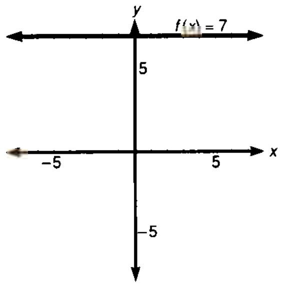
32. y-intercept, -40; x-intercepts, -8 and 5; parabola opening up

33. y-intercepts, 2 and -2; x-intercepts, $2\sqrt{2}$ and $-2\sqrt{2}$; ellipse

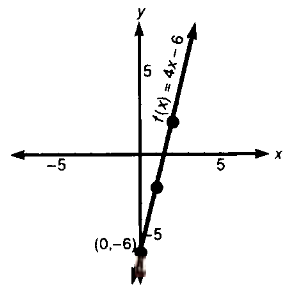
34. x-intercepts, 4 and -4; no y-intercepts; hyperbola

35. x-intercepts, $(-10,0)$, $(2,0)$; y-intercept, $(0,20)$; parabola opening down

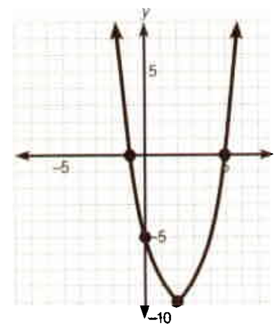
36. constant (linear)



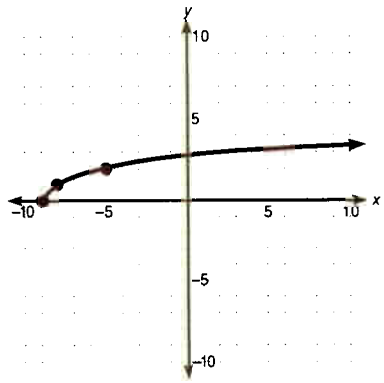
37. linear



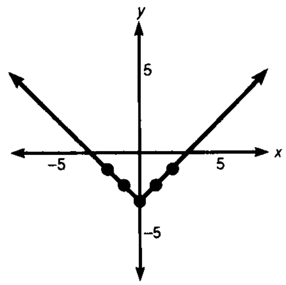
38. quadratic



39. square root



40. absolute value



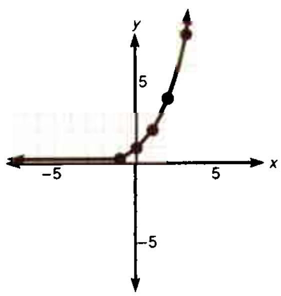
41. a. $k = 32$ **b.** $z = \sqrt[3]{4}$

Chapter 11

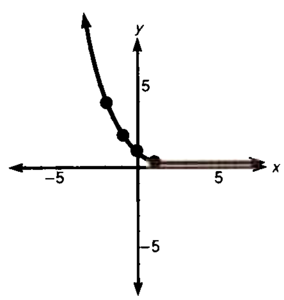
Exercise 11–1

Answers to odd-numbered problems

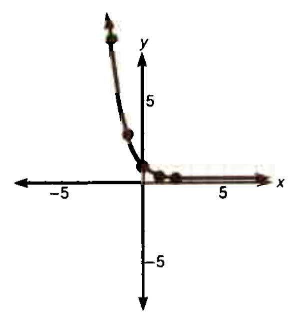
1.



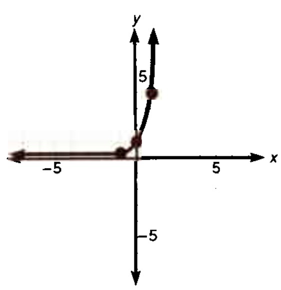
3.



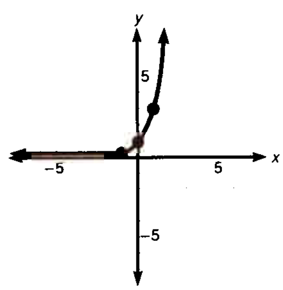
5.



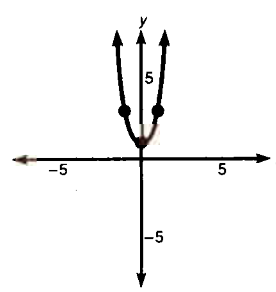
7.



9.



11.



8.66% APR is based on a Prime Rate of 8.25% as of August 1, 2006.

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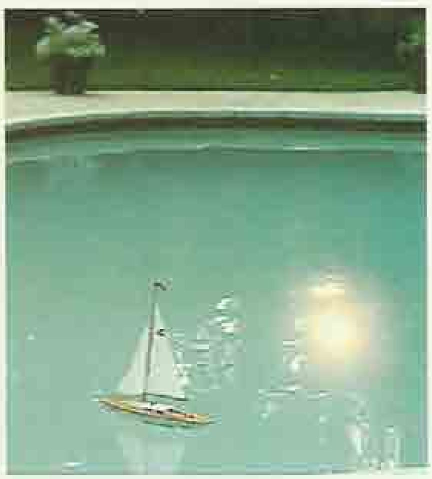

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